ERRATA

In Mr. ATWOOD's *Disquisition on the Stability of Ships.*

Page 213, lines 8 and 9, *dele* " situated between wind and water, according to a technical expression."

Page 223, l. 22, *for* $+$ SB, *read* $+\frac{\mathrm{SB}}{3}$.

—— 247, l. 9, *for* $\frac{\text{Flu. } \dot{x} \times \overline{y^3+p^3}}{3p}$, *read* fluent of $\dot{x} \times \frac{\overline{y^3+v^3}}{p}$.

—— 252, l. 25, *after* point V, *add* produce XL *in directum*, and in the line so produced.

—— 254, l. 22, after the words " proportion of 3 to 5," instead of the *comma* insert a *colon*.

Page 268, l. 8, *for* DD′G′GA, *read* DD′G′GD.

PHILOSOPHICAL

TRANSACTIONS.

X. *A Disquisition on the Stability of Ships. By* George Atwood, *Esq. F. R. S.*

Read March 8, 1798.

THE stability of vessels, by which they are enabled to carry a sufficient quantity of sail, without danger or inconvenience, is reckoned amongst their most essential properties; although the wind may, in one sense, be said to constitute the power by which ships are moved forward in the sea, yet, if it acts on a vessel deficient in stability, the effect will be to incline the ship from the upright, rather than to propel it forward: stability is therefore not less necessary than the impulses of the wind are, to the progressive motion of vessels. This power has also considerable influence in regulating the alternate oscillations of a ship in rolling and pitching; which will be smooth and equable, or sudden and irregular, in a great measure, according as the stability is greater or less at the several angles of inclination from the upright. From constantly observing that the performance of vessels at sea depends materially on their stability, both navigators and naval architects must, at all

times, be desirous of discovering in what particular circumstances of construction this property consists, and according to what laws the stability is affected by any varieties that may be given to their forms, dimensions, and disposition of contents; which are determined partly according to the skill and judgment of the constructor, and partly by adjustments after the vessel has been set afloat.

Little more than a century has now elapsed, since the theory of mechanics was first applied to the construction and management of vessels; whatever principles had been previously adopted, for regulating their forms and equipment, as well as for directing them in the ocean, were the result of experience and observation alone: a mode of arriving at truth, however advantageous in many respects, yet not entirely to be relied on in this instance, for explaining satisfactorily, and reducing to system, phenomena depending on the intricate combination of causes which influence a vessel's motion, and equilibrium, at sea. The theory of mechanics is known to explain all effects that can arise from the action of forces, however complicated, of which the quantities and directions are defined with sufficient precision. This science, having been greatly extended, and successfully employed, by Sir ISAAC NEWTON, in the investigation of causes requiring the most profound research, would naturally be resorted to, for a solution of many difficulties that occur in the theory of naval architecture, which could not be obtained from any other mode of considering this subject. The practice of ship building having been many ages antecedent to the discovery of the theory of mechanics, one object of theoretic inquiry must necessarily be, to explain the principles of construction and management which experience and practical

observation have previously discovered; distinguishing those which are founded in truth and right practice, from others which have been the offspring of vague and capricious opinion, misinterpretation of facts, and unfounded conjecture, by which, phenomena arising in the practice of navigation are often attributed to causes entirely different from those by which they are really governed. It is also the object of mechanic theory to investigate, from the consideration of any untried plans of construction, what will be the effect thereof on the motion of vessels at sea; also to suggest new combinations, by which the approved qualities of vessels may be extended, their faults amended, or defects supplied. These several objects, and others connected with them, have employed the attention of many eminent theorists, by whose discoveries naval architecture has been greatly benefited; yet the progress made toward establishing a general theory, founded on the laws of motion, has not been adequate to what might be expected from the abilities of the writers on this subject, and the laborious attention they have bestowed upon it. Although all results deduced by strict geometrical inference from the laws of motion, are found, by actual experience, to be perfectly consistent with matter of fact, when subjected to the most decisive trials, yet, in the application of these laws to the subject in question, difficulties often occur, either from the obscure nature of the conditions, or the intricate analytical operations arising from them, which either render it impracticable to obtain a solution, or, if a result is obtained, it is expressed in terms so involved and complicated, as to become in a manner useless, as to any practical purpose. These imperfections in the theory of vessels, are amongst the causes which have contributed to

retard the progress of naval architecture, by increasing the hazard of failure in attempting to supply its defects by experiment; for, when no satisfactory estimate can be formed from theory, of the effects likely to ensue from adopting any alteration of construction that may be proposed, doubts must necessarily arise respecting its success or failure, which can be resolved only by having recourse to actual trial: a species of experiment rarely undertaken under the impressions of uncertain success, when the objects of it are so costly, and otherwise of so much importance. To the imperfections of theory, may also be attributed that steady adherence to practical methods, rendered familiar by usage, which creates a disposition to reject, rather than to encourage, proposals of innovation in the construction of vessels: the defects or inconveniences which are known, and have become easily tolerable by use, or may perhaps be the less distinctly perceived for want of comparison with more perfect works of art, being deemed preferable to the adoption of projected improvements, attended by the danger of introducing evils, the nature and extent of which cannot be fully known. These are amongst the difficulties and disadvantages which have concurred in rendering the progress of improvement, in the art of constructing vessels, extremely slow, and have left many imperfections in this practical branch of science, which still remain to be remedied. In respect to the theory of vessels, it would be giving that term too narrowed a meaning, to consider it as derived solely from the laws of mechanics; every notion or opinion which may be applied to explain satisfactorily the phenomena depending on a vessel's construction and qualities, so as to infer the consequences of given conditions, independently of actual trial, whether it ori-

ginates from experience alone, or from investigations founded on the laws of motion, is to be regarded as forming a part of this theory, in which, a constant reference to practice is so essentially necessary. For, although many principles are deducible from the laws of mechanics, which it is probable that no species of experiment, or series of observation, however long continued, would discover, yet there are others, no less important, which have been practically determined with sufficient exactness, the investigation of which it is scarcely possible to infer from the laws of motion; the complicated and ill defined nature of the conditions, in particular instances, rendering analytical operations founded on them liable to uncertainty. Since the practice of naval architecture depends so materially on the knowledge of the causes which influence the motion of vessels at sea, much benefit may probably be derived from the extension of well founded principles, both by attentive observation of the qualities of vessels, compared with their construction, as well as by investigation of the effects arising from particular modes of construction, depending on the laws of statics and mechanics, whenever the conditions admit of inferring principles which are clear and satisfactory, and easily applicable in practice. With a view to these objects, so far as regards the theory of stability, the ensuing Disquisition has been written.

When a ship, or other floating body, is deflected from its quiescent position, the force of the fluid's pressure operates to restore the floating body to the situation from which it has been inclined. This force is distinctly described, in a treatise written by the most celebrated geometrician of ancient times, who uses the following argument for demonstrating the position in which a parabolic conoid will float permanently in given cir-

cumstances. To shew that this solid will float with the axis inclined to the fluid's surface at a certain stated angle, depending on the specific gravity and dimensions of the solid, he demonstrates,* that if the angle should be greater than that which he has assigned, the fluid's pressure will diminish it; and that, if the angle should be less, the fluid's pressure will operate to increase it, by causing the solid to revolve round an axis which is parallel to the horizon. It is an evident consequence, that the solid cannot float quiescent with the axis inclined to the fluid's surface, at any angle except that which is stated. The force which is shewn in this proposition, to turn the solid, so as to alter the inclination of the axis to the horizon, is the same with the force of stability; the quantity or measure of which, ARCHIMEDES does not estimate; nor was it necessary to his purpose, since the alteration of inclination required to establish the quiescent position, may be produced either in a greater or less time, without affecting his argument. It does not appear, that this method of determining the floating positions of bodies was afterwards extended to infer similar conclusions in respect to solids of any other forms, nor to determine any thing concerning the inclination or equilibrium of ships at sea, which require the demonstration, not only that a force exists, in given circumstances, to turn the vessel round an axis, but also the magnitude or precise measure of that force. M. BOUGUER, in his treatise intitled "*Traitè du Navire,*" † has investigated a theorem for estimating the exact measure of the stability of floating bodies. This theorem, in one sense, is general, not being confined to bodies of any particular form; but, in respect to the angles of

* ARCHIMEDES *de iis quæ in humido vehuntur.* † *Livr.* ii. *sect.* 2. *chap.* 8.

inclination, it is restrained to the condition that the inclinations from the upright shall be evanescent, or, in a practical sense, very small angles. In consequence of this restriction, the rule in question cannot be generally applied to ascertain the stability of ships at sea; because the angles to which they are inclined, both by rolling and pitching, being of considerable magnitude, the stability will depend, not only on the conditions which enter into M. BOUGUER's solution, but also on the shape given to the sides of the vessel above and beneath the water-line or section, of which M. BOUGUER's theorem takes no account. But it is certain that the quantity of sail a ship is enabled safely to carry, and the use of the guns in rough weather, depend in a material degree on the form of the sides above and beneath the water-line; this observation referring to that portion of the sides only which may be immersed under, or may emerge above, the water's surface, in consequence of the vessel's inclination; for, whatever portion of the sides is not included within these limits, will have no effect on the vessel's stability, the centres of gravity, volume of water displaced, and other elements not being altered. By the water-section is meant, the plane in which the water's surface intersects the vessel, when floating upright and quiescent; and the termination of this section in the sides of the vessel is termed the water-line. A general theorem for determining the floating positions of bodies is demonstrated in a former paper, inserted in the Phil. Trans. for the year 1796, and applied to bodies of various forms: the same theorem is there shewn to be no less applicable to the stability of vessels, taking into account the shape of the sides, the inclination from the upright, as well as every other circumstance by which the

stability can be influenced. To infer, from this theorem, the stability of vessels in particular cases, the form of the sides, and the angle of inclination from the perpendicular, must be given. These conditions admit of great variety, considering the shape of the sides, both above the water-line and beneath it; for we may first assume a case, which is one of the most simple and obvious; this is, when the sides of a vessel are parallel to the plane of the masts, both above and beneath the water-line; or, secondly, the sides may be parallel to the masts under the water-line, and project outward, or may be inclined inward, above the said line; or they may be parallel to the masts above the water-line, and inclined either inward or outward beneath it; some of these cases, as well as those which follow, being not improper in the construction of particular species of vessels, and the others, although not suited to practice, will contribute to illustrate the general theory. The sides of a vessel may also coincide with the sides of a wedge, inclined to each other at a given angle; which angle, formed at an imaginary line, where the sides, if produced, would intersect each other, may be situated either under or above the water's surface. To these cases may be added, the circular form of the sides, and that of the Apollonian or conic parabola. The sides of vessels may also be assumed to coincide with curves of different species and dimensions, some of which approach to the forms adopted in the practice of naval architecture, particularly in the larger ships of burden. And lastly, the shape of the sides may be reducible to no regular geometrical law; in which case, the determination of the stability, in respect to a ship's rolling, requires the mensuration of the ordinates of the vertical sections which intersect the longer

axis at right angles; similar mensurations are also required for determining the stability, in respect to the shorter axis, round which a vessel revolves in pitching. In order to describe distinctly these several cases, the variation of the sections, both in form and magnitude, from head to stern of the vessel, has not been considered; the sections being supposed equal and similar figures, such as they in reality are, near the greatest section of a ship, growing smaller, and altering their form, toward the head and stern. But, before this alteration can be taken into account, it is necessary first to ascertain the stability corresponding to a vessel or segment, in which the sections are equal and similar figures; from which determination, the stability is inferred which actually exists, when the form and magnitude of the sections alter continually, from one extremity of the vessel to the other. The consideration of the cases which have been here stated, with inferences and observations thereon, is the subject of the ensuing pages; in which, if any ideas are suggested which may be at all useful in the practice of naval architecture, or may contribute to remove imperfect or erroneous notions which have been entertained respecting a principal branch of it, the intention of the Author will be accomplished.

Let WBCOFAH (Tab. VIII. fig. 1.) represent a vertical section of a vessel floating quiescent and upright, and intersected by the water's surface in the line BA: BCOFA will be the area immersed under water. Suppose the vessel to be inclined from the perpendicular, through the angle ASH, so that the intersection of the vessel by the water's surface, which before coincided with BA, shall now coincide with the line

CH: the area under water will now be COFAH, equal to the area BCOFA.

Let the section WBCOFAH, and all the other vertical sections intersecting the longer axis at right angles, be assumed similar and equal figures, projected on the plane WBOAH: in consequence, the area BOA will be to the area ASH, as the entire volume immersed is to the volume immersed by the vessel's inclination. Moreover, if E is the centre of gravity of the area BOA, that point will truly represent the centre of gravity of the volume immersed, when the vessel is upright: if the centre of gravity of the immersed area COFAH, when the vessel is inclined, should be situated at Q, that point will also coincide with the centre of gravity of the corresponding displaced volume. For these reasons, the spaces BOA, ASH, COFAH, will be denominated, in the following pages, indifferently, areas or volumes.

Let G be the centre of gravity of the vessel, by which term, the vessel and its contents, of every kind, are always understood to be implied. Through G, draw GU parallel to CH: and through Q, draw QZ perpendicular to CH. When the ship is inclined round the longer axis, through the angle ASH, the fluid's pressure acts in the direction of the vertical line QZ, with a force equal to the vessel's weight; and the stability or effect of this force, to turn the vessel round an axis passing through G, perpendicular to the plane BOA, will be greater or less, according to the magnitude of the line GZ, or distance from the axis at which the force of pressure acts. In the same vessel, the weight not being altered, the stability, at different angles of inclination from the upright, will be truly measured by the line GZ; and, in different vessels, or in the

same vessel differently laden, the stability will be measured by the weight of the vessel and the line G Z jointly. The weight of any vessel (including the lading) is equal to the weight of water displaced by it; which will be obtained by measuring the solid contents of the displaced volume, and from knowing the weight of a given portion of sea water, such as a cubic foot, which weighs 64 pounds avoirdupois. The vessel's weight being thus obtained, the determination of the stability, whatever be its form or inclination from the upright, requires only that the line G Z shall be known, or the proportion which it bears to some given line, for instance, the line B A, shall be ascertained.

A general method of constructing this line is demonstrated in the Phil. Trans. for the year 1796, but is there principally applied to the floating position of bodies; its use in investigating the stability of vessels is incidentally mentioned, and in general terms, rather than as being itself a subject of disquisition. This theorem is founded on supposing the centres of gravity of the several volumes BOA, COFH, ASH, BSC, (fig. 1.) to be given in position; an assumption allowable in demonstrating a general theorem: but, in applying it to the stability of particular vessels, it becomes necessary that the positions of these points should be absolutely found, and the results combined with the other conditions, to infer the measure of stability; a determination which, in some cases, is attended with much difficulty, and in others, is not practicable by any direct methods; an instance, amongst many that might be mentioned, in which the particular application is more difficult than the general demonstration of propositions. The following constructions and investigations are

principally inferred from the general theorem for ascertaining the stability of floating bodies; which is here subjoined, to avoid the necessity of future references, as well as for the purpose of stating more distinctly the observations which follow it.

Let M (fig. 1.) be the centre of gravity of the volume ASH, which has been immersed under water, and let I be the centre of gravity of the volume BSC, which has emerged above the water's surface, in consequence of the vessel's inclination; through the points M and I, draw the lines ML, IK, perpendicular to the line CH, which coincides with the water's surface when the vessel is inclined: through E, the centre of gravity of the displaced volume BOA, draw EV parallel and equal to KL, and through G draw GU parallel and GR perpendicular to CH; according to the theorem, the line ET will be determined by the following proportion. As the total volume displaced BOA is to SAH, the volume immersed in consequence of the inclination, so is KL or EV to ET; and, since the angle EGR is equal to the vessel's inclination ASH, and the distance GE is supposed to be given, the line ER will be known; because ER is to GE as the sine of the angle EGR to radius; ER being subtracted from ET will leave RT or GZ, equal to the measure of the vessel's stability.

Suppose the line KL to be denoted by the letter b: let the volume ASH be represented by A, and the volume BOA by V. Then, according to the theorem, since $V : A :: b :$ ET, it follows that $ET = \frac{bA}{V}$, and if GE is put $= d$, and s $=$ the sine of the angle to which the vessel is inclined, radius being $= 1$, ER will be $= ds$; and the measure of the vessel's stability RT or $GZ = \frac{bA}{V} - ds$.

Through the points C and H, (fig. 1.) let the lines CF, WH, be drawn parallel to BA. The position of the points M and I, the magnitude of the line KL, and the areas or volumes ASH, BSC, being the same, whatever alteration may take place in the volume V, or the entire volume displaced, the quantity KL $\times$ area ASH or bA will remain the same: and, since the line ET $= \frac{bA}{V}$, it will follow, that the zone WHFC, situated between wind and water, (according to a technical expression,) not being altered, ET will be in the inverse proportion of V, or the total volume displaced. If, therefore, the shape of the vessel under the line CF should be any how changed, so as to coincide with another figure, suppose CcfF, (fig. 2.) instead of COF, (fig. 1.) the volume CcfF being equal to the volume COF, the line ET will be the same in both cases. In consequence of this change of figure, the position of the point E, (fig. 1.) or centre of gravity of the volume BOA, may be situated higher or lower in the line OD; yet, if the centre of gravity G is so adjusted by ballast, or other means, that the distance GE shall be the same, the stability of each vessel, BCOA (fig. 1.) and BCcfA (fig. 2.) will be perfectly the same, when inclined to the same angle ASH from the upright. It must also be observed, that since ET is always greater in the same proportion in which the volume immersed BOA is less, the zone WHCF being both in magnitude and form the same, having found by construction or calculation the value of the line ET corresponding to any given volume displaced, suppose V = BCOA, (fig. 1.) the line Et corresponding to any other magnitude of volume displaced, suppose v = BCVwlFA, (fig. 2.) will be immediately inferred; for, since V . v : : Et : ET, it

follows that $E\,t = \frac{E\,T \times V}{v}$, or because $E\,T = \frac{b\,A}{V}$, by substitution, $E\,t = \frac{b\,A}{v}$. For these reasons, the determination of stability does not require that the form of the entire volume displaced should be given, but the form only of the zone W C H F, (fig. 1. and 2.) including the angle of the vessel's inclination A S H; these conditions, together with the magnitude of the immersed volume, and the distance between the two centres of gravity G and E, are sufficient for finding the measure of stability, at any given angle of inclination from the upright.

CASE I.

The sides of a vessel are parallel to the plane of the masts, both above and beneath the water-line.

Q B C O A H (fig. 3) coincides with the vertical section of a vessel when it floats upright and quiescent, and is intersected by the water's surface in the line B A; the sides Q C, H D, are parallel to each other, and to the plane of the masts W O, and are therefore perpendicular to B A. G is the centre of gravity of the vessel; V represents the magnitude of the volume immersed under the water; the centre of gravity of this volume is situated at E. Suppose the vessel to be inclined from its quiescent position through any given angle, it is required to express, by geometrical construction, the measure of the vessel's stability, when thus inclined. Bisect B A in the point S, and through S draw C S H, inclined to B A, at the given angle of the vessel's inclination from the upright. Bisect BC in F, and AH in N; and join SF and SN. In the line SF take SI to SF as 2 to 3; also, in the line SN, take

SM to SN as 2 to 3. Through the points I and M, draw IK, ML, perpendicular to CH. Through the point E, draw EV parallel and equal to KL. In the line EV, take ET to EV, in the proportion which the volume ASH bears to the entire volume displaced. Through G, draw GU parallel to CH; and through T, draw TZ perpendicular to GU. GZ is the measure of the vessel's stability. The demonstration of this construction evidently follows from the general theorem.

From this construction, the value of GZ, or measure of the vessel's stability, may be investigated analytically, and expressed in general terms. Through G, draw GR perpendicular to EV. Let $BA = t$, $GE = d$, the angle $ASH = S$; radius $= 1$. The rules of trigonometry give the following determinations. $AN = \frac{t \times \text{tang.} S}{4}$: $SN = \frac{t}{4} \times \sqrt{4 + \text{tang.}^2 S}$. Also, as $SN : HN :: \text{sine } NHS : \text{sin. } NSH$, or $\frac{t}{4} \times \sqrt{4 + \text{tang.}^2 S} : \frac{t \times \text{tang.} S}{4} :: \text{cos.} S : \text{sin.} NSH$. Wherefore $\text{sin.} NSH = \frac{\text{sin.} S}{\sqrt{4 + \text{tang.}^2 S}}$; $\text{cos.}^2 NSH = \frac{4 + \text{tang.}^2 S - \text{sin.}^2 S}{4 + \text{tang.}^2 S} = \frac{2 + \text{sec.}^2 S + \text{cos.}^2 S}{4 + \text{tang.}^2 S} = \frac{\overline{\text{sec.} S + \text{cos.} S}|^2}{4 + \text{tang.}^2 S}$ (because $2 \times \text{cos.} S \times \text{sec.} S = 2$) consequently $\text{cos.} NSH = \frac{\text{sec.} S + \text{cos.} S}{\sqrt{4 + \text{tang.}^2 S}}$. And since by construction, $SM = \frac{2}{3} SN$, and $SN = \frac{t}{4} \times \sqrt{4 + \text{tang.}^2 S}$, $SM = \frac{t}{6} \times \sqrt{4 + \text{tang.}^2 S}$, and $SL = \frac{t}{6} \times \sqrt{4 + \text{tang.}^2 S} \times \frac{\text{sec.} S + \text{cos.} S}{\sqrt{4 + \text{tang.}^2 S}} = \frac{t}{6} \times \overline{\text{sec.} S + \text{cos.} S}$: and the triangles SLM, SIK being similar and equal, $KL = 2 SL$: Wherefore $KL = \frac{t}{3} \times \overline{\text{sec.} S + \text{cos.} S} = EV$. The area of the triangle $ASH = \frac{t^2 \times \text{tang.} S}{8}$ representing the volume immersed by the vessel's inclination; and by construction,

As V : volume A S H : : E V : E T, or

$V : \frac{t^2 \times \text{tang. S}}{8} : : \frac{t}{3} \times \overline{\text{sec. S} + \text{cos. S}} : ET$; this will give the value of $ET = \frac{t^3 \times \text{tang. S} \times \overline{\text{cos. S} + \text{sec. S}}}{24\, V}$: and because

E R : E G : : sin. S : 1, and E G $= d$, it follows, that E R $= d \times$ sin. S; and therefore R T, or the measure of the vessel's stability $GZ = \frac{t^3 \times \text{tang. S}}{24\, V} \times \overline{\text{cos. S} + \text{sec. S}} - d \times \text{sin. S}$.

To exemplify this determination by referring to a particular case, let the vessel's breadth at the water's surface, or BA, be divided into 100 equal parts, and let G E be 13 thereof; so that $t = 100$, and $d = 13$. Suppose the inclination of the vessel from the perpendicular, or A S H, to be 15°, = S; and let the area B C O D A, representing the volume displaced, be equal to a square of which the side is = 60; so that the area V shall = 3600: then, referring to the solution, we obtain

$$\text{cos. S} + \text{sec. S} = 2.0012$$

$$\text{Also } \frac{t^3 \times \text{tang. S}}{24\, V} = \frac{1000000 \text{ tang. } 15°}{24 \times 3600} = 3.1013$$

$$ET = 2.0012 \times 3.1\dot{0}190 = 6.2063$$

$$d \times \text{sin. S} = 13 \times \text{sin. } 15° = 3.3646$$

$$\text{measure of stability, or } GZ = 2.8417$$

It appears by this result, that when the vessel has been inclined from the upright through an angle of 15°, the direction of the fluid's pressure, acting to restore the quiescent position, will pass at a distance estimated horizontally from the axis = 2.84, when the breadth B A = 100. And this will be true, whatever be the length of the axis.

The fluid's pressure is the weight of water displaced, the magnitude of which depends both on the area of the vertical

sections, and length of the axis: suppose this weight to be 1000 tons; according to the preceding determination, the stability of the vessel, when inclined from the upright to an angle of 15°, will be a pressure equal to the weight of 1000 tons, acting at a distance of $\frac{2,84}{100}$ parts of the breadth B A from the axis, to restore the vessel to the position from which it has been inclined. This force is the same as if a pressure of $\frac{1000 \times 2,84}{50} = 56.8$ tons, should be applied to turn the vessel at the distance of 50 from the axis: if therefore the wind, or other equivalent power, should act on the sails of the vessel with a force of 56.8 tons, at the mean or average distance of 50, or $\frac{1}{2}$ the breadth BA from the axis, to incline the ship, the force of stability will just balance it, so as to preserve an equilibrium; the vessel continuing inclined from the upright at the angle of 15°. If the wind's force should be less, the inclination must necessarily be diminished; if greater, it must be increased, until the two forces balance each other. Here it is to be observed, that the force of the wind is estimated in a direction which is perpendicular to the plane of the masts.*

* In this and the following numerical examples, in order to bring into comparison the effect of giving different forms to the sides of vessels, their weights, and all the other conditions (the figure of the sides excepted) on which the stability depends, are assumed to be the same. The measures of stability are compared, both by the relative distances from the axis at which a given pressure, equal to the vessel's weight, acts to turn the ship round the longer axis, and by the relative equivalent weights which act at a given distance from the axis. By the latter method, the proportions of stability are perhaps more distinctly expressed than by the former, although both are essentially the same.

The mechanical force employed to incline a vessel from the upright, through any given angle, for the purpose of examining and repairing the bottom of a ship, is to be ascertained from the theorems here given for expressing the measures of stability,

CASE II.

The sides of a vessel project outward above the water-line, and are parallel to the masts under the water-line.

The line B A (fig. 4.) represents the intersection of the water's surface with the vessel, when floating upright. The lines PC, QW, are parallel to each other, and to the line XO, which coincides with the plane of the masts, and bisects the line BA in the point D; B C and AW, which are parallel to the plane of the masts, coincide with the sides of the vessel under the water-line; and BY, AH, which project outwards from the plane of the masts, at the angle QAH, or YBP, are the sides of the vessel above the water-line. C H represents the intersection of the water's surface with the vessel, when inclined from the perpendicular, through a given angle OPQ = ASH. The distance G E, between the centres of gravity of the vessel and of the volume displaced, and the magnitude of that volume being supposed known, and the angle QAH, at which the sides AH, BY, are inclined to the plane of the masts, being also known, it is required to ascertain, by geometrical construction, the measure of the vessel's stability, when the

which is exactly equal to the force to be applied for that purpose. Another method of inclining a vessel (well adapted for making experiments on this subject) is, by applying a timber at right angles to the plane of the masts. If a weight be affixed to one of its extremities, from having given the weight so applied, and its distance from the plane of the masts, together with the other conditions which determine stability, the angle of inclination, through which the ship will be inclined, may be determined by the theorems in these pages. The same inferences may be obtained, from having given the weights and spaces through which the guns are run out on one side, and drawn in on the other, instead of the weight affixed, according to the method last described.

vessel is inclined from the perpendicular, through an angle equal to the angle OPQ.

At whatever angle the vessel may be inclined from the perpendicular, the total volume immersed must always remain the same, while the vessel's weight continues unaltered. Wherefore, the volume which has been immersed, or ASH, must be equal to the volume BSC, which has emerged from the water, in consequence of the vessel's inclination. For this reason, and because the side AH projects outward, while the side BC is parallel to the plane of the masts, it must necessarily happen, that the point S will not in this case bisect the line BA, as it did in the preceding construction, but will be removed nearer to the side AH, which has been immersed by the inclination of the vessel. Previously, therefore, to any consideration of the stability, it will be necessary to define the position of the point S in the line AB, so that a line CH, being drawn through S, at a given angle of inclination to AB, equal to that of the ship's inclination from the perpendicular, shall cut off the area ASH equal to the area BSC.

Let the given angle of inclination be OPQ equal to ASH (fig. 4.): the angle QAH, at which the sides of the vessel are inclined outward from the plane of the masts above the water-line, is supposed to be given: this angle + 90° will be the angle SAH, which is therefore a known quantity: the remaining angle SHA, in the triangle ASH, will likewise be known.

Through the extremity B of the line BA, (fig. 5.) equal to the vessel's breadth at the water-line, draw the indefinite line BU inclined to BA, at an angle ABU equal to OPQ: in BU, take any point O, and, in the line BO, set off BD to BO, as the cosine of the angle ABU to radius. In the line BD, take BE to BD, as the sine of the angle BAU is to radius: also take BF to

BO, as the sine of the angle AUB to radius. Let BG be taken a geometrical mean proportional between the lines BF and BE; and from the point G, in the direction of the line GU, set off GZ equal to BF: join AZ; and, through the point G, draw GS parallel to ZA, intersecting BA in the point S. Through S, draw the line CH parallel to BU: the area ASH will be equal to the area BSC.

Since, in the triangles ASH, BSC, the angle ASH is equal to the angle BSC, the areas of the triangles will be in a ratio compounded of the ratios of the sides, including the equal angles; that is, the area of the triangle ASH, will be to the area of the triangle BSC in the ratio of SA $\times$ SH to SB $\times$ SC. By the construction, the angle ASH = the angle ABU = OPQ; and the angle AHS = the angle AUB: also, by construction,

BO : BD : : rad. : cos. ASH.
Also BD : BE : : rad. : sin. SAH.
And BF : BO : : sin. AHS : rad.

Joining these ratios BF : BE : : sin. AHS $\times$ rad. : cos. ASH $\times$ sin. SAH.

But, by the construction, and by the similarity of the triangles, BGS, BZA, BF or GZ : BE* : : $BF^2 : BG^2$: : $GZ^2 : BG^2$: : $SA^2 : SB^2$:

Wherefore $SA^2 : SB^2$: : sin. AHS $\times$ rad. : cos. ASH $\times$ sin. SAH.
And by trigonometry SH : SA : : sin. SAH : sin. AHS;
and SB : SC : : cos. ASH : rad.

Joining these ratios SA $\times$ SH : SB $\times$ SC : : rad. : rad.

But the area ASH is to the area BSC as SA $\times$ SH to SB $\times$ SC; consequently, the area ASH is equal to the area BSC.

To proceed with the construction of the second case. Through the point S, (fig. 4.) determined by the preceding construction, draw the line CH inclined to BA at the angle

* Because the ratio of BE to BG is equal to the ratio of BG to BF, by the construction, it will follow that the ratio of BE to BF is double the ratio of BE to BG.

ASH, equal to the given angle OPQ: when the vessel is inclined from the perpendicular through this angle, it will be intersected by the water's surface coinciding with the line CH. Bisect BC in F, and AH in N; and join SF, SN: take SI to SF, as 2 to 3; and SM to SN, in the same proportion. Through I and M, draw the lines IK, ML, perpendicular to CH. Through the centre of gravity of the vessel G, draw GU parallel to CH; and through the centre of gravity E, of the displaced volume BOA, draw EV parallel and equal to KL; and in EV take ET to EV, in the same proportion which the volume ASH bears to the entire volume displaced BOA. Through T, draw TZ perpendicular to GU. GZ is the measure of the vessel's stability.

To obtain an analytical value of the line GZ, for brevity, let the sine of the angle ASH be denoted by s, when radius is $= 1$, make sin. HAS $= a$; sin. AHS $= b$; sin. SCB $= c$. Let GE $= d$. Also, let the entire volume displaced $=$ V. By the rules of trigonometry, it is found that

$$* \; \mathrm{SL} = \frac{\mathrm{SA}}{3} \times \sqrt{4 + s^2 \times \frac{\overline{1 - b^2}}{b^2} + \frac{4s \times \sqrt{1 - a^2}}{b}}$$

$$\text{Also SK} = \frac{\mathrm{SB}}{3} \times \overline{\cos.\,\mathrm{ASH} + \sec.\,\mathrm{ASH}}.$$

$$\text{The area SCB or ASH} = \frac{\overline{\mathrm{SB}^2} \times \text{tang. ASH}}{2}$$

$$\text{Wherefore ET} = \frac{\mathrm{SA} \times \mathrm{SB}^2 \times \text{tang. ASH}}{6\mathrm{V}} \times \sqrt{4 + s^2 \times \frac{\overline{1 - b^2}}{b^2} + \frac{4s \times \sqrt{1 - a^2}}{b}} +$$

$+ \dfrac{\mathrm{SB}^3 \times \text{tang. ASH}}{6\mathrm{V}} \times \overline{\cos.\,\mathrm{ASH} + \sec.\,\mathrm{ASH}}.$ If the breadth BA

* When the angle SAH $= 90$, $1 - a^2 = 0$; and $b = \cos.\,\mathrm{S}$; in which case, if c is put $= \cos.\,\mathrm{S}$, $\mathrm{SL} = \dfrac{\mathrm{SA}}{3} \times \sqrt{4 + \dfrac{s^4}{c^2}}$, but $4 + \dfrac{s^4}{c^2} = 4 + \text{tang.}^2\,\mathrm{S} \times \sin.^2\,\mathrm{S} = 4 + \text{tang.}^2\,\mathrm{S} - \text{tang.}^2\,\mathrm{S} \times \cos.^2\,\mathrm{S} = 2 + \sec.^2\,\mathrm{S} + \cos.^2\,\mathrm{S} = \overline{\cos.\,\mathrm{S} + \sec.\,\mathrm{S}}|^2$. Wherefore $\mathrm{SL} = \dfrac{\mathrm{SA}}{3} \times \overline{\cos.\,\mathrm{S} + \sec.\,\mathrm{S}}$.

be represented by the letter t, it is inferred, from the construction in p. 220, that $SA = \frac{t \times \sqrt{b}}{\sqrt{b} + \sqrt{ac}}$ and $SB = \frac{t \times \sqrt{ac}}{\sqrt{b} + \sqrt{ac}}$. The value of the line ET having been thus determined, if $ER = d \times$ sin. ASH or $d\,s$ be subtracted from it, the result will be GZ, the measure of the vessel's stability.

Suppose the sides BY, AH, (fig. 4.) to project outward, at an angle of 15° inclination to the parallel sides BC, AW, so as to make the angle SAH = 105°. Let the vessel's inclination from the upright be the angle ASH = 15°; and therefore AHS = 60°, and SCB = 75°. Let the breadth BA or t = 100 equal parts, of which d or GE = 13. Then, by calculating from the analytical values just determined, it is found that KL = SL + SK = 68,017: the area ASH = 347.44, and the entire volume immersed V, being, as in the former case, = 3600, ET = $\frac{68.017 \times 347.44}{3600} = 6.57$. And, since ER or $d \times$ sin. ASH is = 3.36, if the latter value be subtracted from the former, the result will be GZ = 3.21, or the measure of the vessel's stability.

The force of stability, to restore the vessel to the upright position, will be precisely the vessel's weight, or fluid's pressure, acting in the direction of a vertical line, which passes at a distance of 3.20 from the axis, estimated in a horizontal direction. And this force is equivalent to, and will counterbalance, $\frac{3.21}{50}$ parts of the vessel's weight, applied to act in a contrary direction, at the distance of 50 from the said axis. So that, if the vessel's weight should be 1000 tons, the force of stability would balance a weight or force of $\frac{3.21 \times 1000}{50} = 64.2$ tons, applied to act at the distance 50 from the axis.

CASE III.

The sides of a vessel are inclined inward above the water-line, and are parallel to the plane of the masts under the water-line.

AH, BY, (fig. 6.) are the sides of a vessel inclined inward above the water-line BA, at an angle HAQ = YBP from the direction of the sides AW, BC, under the water-line, which are parallel to each other, and to the plane of the masts. Suppose the vessel to be inclined from the upright, through an angle = OPQ. By the construction, (p. 220.) draw the line CH intersecting BA, in a point S, at an angle ASH equal to the given angle OPQ; so that the area ASH shall be equal to the area BSC. When the vessel has been inclined through the given angle OPQ, it will be intersected by the water's surface in the line CH. The construction of the line GZ, or measure of the vessel's stability, is the same as in the preceding case.

Let the sine of ASH = s; sin. SAH = a; sin. SHA = b; sin. SCB = c to rad. = 1. Also let GE = d.

From the rules of trigonometry, it is inferred that

$$KL = \frac{SA}{3} \times \sqrt{4 + s^2 \times \frac{1-b^2}{b^2} - \frac{4s \times \sqrt{1-a^2}}{b}}$$

$$+ SB \times \overline{\cos. ASH + \sec. ASH}.$$

The area SBC or ASH = $\frac{SB^2 \times \text{tang. } ASH}{2}$.

If, therefore, the total volume immersed is made = V, the value of the line ET will be

$$ET = \frac{SA \times SB^2 \times \text{tang. } ASH}{6V} \times \sqrt{4 + s^2 \times \frac{1-b^2}{b^2} - \frac{4s \times \sqrt{1-a^2}}{b}}$$

$+\frac{SB^3 \times \text{tang. } ASH}{6V} \times \overline{\text{cos. } ASH + \text{sec. } ASH}$; in which expression $SA = \frac{t \times \sqrt{b}}{\sqrt{b} + \sqrt{ac}}$, and $SB = \frac{t \times \sqrt{ac}}{\sqrt{b} + \sqrt{ac}}$; t being $=$ the breadth BA.

The value of ET having been thus obtained, if $ER = d \times$ sine ASH be subtracted from it, there will remain the value of GZ, the measure of the vessel's stability.

Suppose the vessel's inclination from the perpendicular, or ASH, to be $= 15°$, let the inclination of the sides inward above the water-line, from the direction of the parallel sides under the water, or HAQ $= 15°$; therefore SAH $= 75°$, and SHA $= 90°$, making BA $= t$, and, applying these conditions to the analytical value just determined, it is found that KL $= 65.530$; the area ASH $= 323.42$; and the volume immersed, or V, being assumed $= 3600$, as in the preceding cases, $ET = \frac{65.524 \times 323.42}{.3600}$ $= 5.89$. Subtracting from this, ER $= 3.36$, there will remain GZ $= 2.53$, or the measure of stability. If the vessel's weight should be 1000 tons, the force of stability will be 1000 tons, acting to turn the vessel at a distance of $\frac{2.53}{100}$ parts of the breadth BA from the axis; which is equal to a force or weight of $\frac{1000 \times 2.53}{50} = 50.6$ tons, acting to turn the vessel at a distance of 50 from the axis.

CASE IV.

The sides of a vessel project outwards, and at equal inclinations to the plane of the masts, both above and beneath the water-line.

BA (Tab. IX. fig. 7.) is the breadth of the vessel, and coin-

cides with the water's surface, when the vessel floats upright. XE denotes the plane of the masts, bisecting BA in the point S. PU, QW, are lines drawn through the extremities of the line BA, and perpendicular to it, and therefore parallel to EX: the sides of the vessel above the water-line, AH, BY, are inclined outward from the plane of the masts, at an angle QAH = PBY; and BC, AD, are the sides under the water-line, also inclined outward from the plane of the masts, at an angle = DAW = CBU = QAH. G and E represent the centres of gravity of the vessel, and of the volume displaced, as in the former cases. To construct the measure of stability, corresponding to any given angle of inclination from the upright,

Through the point S, which bisects the line BA, draw the line CH inclined to BA, at the angle ASH, equal to the given angle of inclination from the upright. Since, by the conditions of this problem, the triangles ASH, BSC, are similar and equal figures, it follows, that when the vessel is inclined from the perpendicular through the angle ASH, it will be intersected by the water's surface, in the direction of the line CH. The subsequent part of this construction is similar to those of the preceding cases, as sufficiently appears by inspection of the figure.

Let the breadth of the vessel at the water's surface, or BA $= t$: put the sine of the angle ASH $= s$, sine SAH $= a$, sine SHA $= b =$ radius $= 1$, GE $= d$. Then the area ASH, or BSC $= \frac{t^2 sa}{8b}$, and, if the total volume immersed is put $=$ V, the measure of the vessel's stability, or GZ, will be $= \frac{t^3 sa}{24 Vb} \times$

$$\sqrt{4 + s^2 \times \frac{1-b^2}{b^2} + \frac{4s \times \sqrt{1-a^2}}{b}} - d s.$$

Let BA, or $t = 100$, $d = 13$, ASH $= 15°$, SAH $= 105°$, V $= 3600$, as in the former cases: then, $s =$ sine $15°$, $a =$ sine $105°$, $b =$ sine $60°$; by referring to the solution, GZ $= 3.59$; and the stability will be the weight of the vessel, suppose 1000 tons, acting at the distance 3.59 from the axis, to turn the vessel; which force is equivalent to a weight of 71.7 tons, applied at the distance of 50 from the axis.

CASE V.

The sides of a vessel are inclined inward, and at equal angles of inclination to the plane of the masts, both above and beneath the water-line.

BA (fig. 8.) is the breadth of the vessel coinciding with the water's surface, when floating upright. XE represents the plane of the masts, bisecting BA in the point S. UP, WQ, are lines drawn through the extremities of the line BA, parallel to XE. BY, AH, are the sides of the vessel above the water-line, inclined inward to the plane of the masts, at the angle QAH = YBP. BC, AD, are the sides under the water-line, inclined inward to the plane of the masts, at the angle DAW or CBU, which are equal to HAQ or YBP. The other conditions are as in the former cases. Through the point S, draw the line CH inclined to BA, at the angle ASH, equal to the vessel's inclination from the upright. Since the triangles ASH, BSC, are similar and equal figures, it follows, that when the vessel is inclined to the angle ASH, it will be intersected by the water's surface in the line CH. The remaining part of this construction is similar to that of the preceding cases.

The same notation being adopted with that which was used

in the preceding case, by referring to trigonometrical properties, it is found that the measure of stability, or

$$GZ = \frac{t^3 sa}{24Vb} \times \sqrt{4 + s^2 \times \frac{\overline{1-b^2}}{b^2} - \frac{4s \times \sqrt{1-a^2}}{b}} - ds.$$

Let $t = 100$, $d = 13$, the inclination of the sides inward, or HAQ $= 15°$, ASH $= 15°$, SAH $= 75°$, SHA $= 90°$: by calculating from these data, it is found that GZ $= 2.21$.

If the vessel's weight should be 1000 tons, the stability will be this weight, acting to turn the vessel at the distance 2.21 from the axis; which is equivalent to a force of 44.2 tons, applied at the distance of 50 from the axis.

CASE VI.

The sides of a vessel coincide with the sides of an isosceles wedge, (fig. 9.) meeting, if produced, in an angle BWA, which is beneath the water's surface.

Supposing the sides to be continued till they meet, the vertical sections will be equal isosceles triangles. BAW represents one of these triangles, BA being coincident with the water's surface, and cutting off the line BW equal to AW. The angle WBA = WAB is supposed to be given. If the vessel should be inclined from the perpendicular, so that the water's surface shall coincide with the line CH, the point of intersection S must be so situated, that the area or volume immersed, in consequence of the inclination, that is, ASH, shall be equal to the area or volume SBC, which has emerged from the water. Previously, therefore, to the construction of this case, the position of the point S is to be geometrically determined, according to the conditions required.

Let BWA (fig. 10.) represent a vertical section of the vessel. Through the extremity B of the line BA, draw BO inclined to BA, at the angle ABO, equal to the vessel's inclination from the upright. In this line, take any point R, and in BR take BI to BR, as the sine of the angle WBR to radius. Also take BF to BR as the sine of BRW to radius; and let BG be a geometrical mean proportional between the lines BF and BI; from the point G, set off GZ equal to BF; join ZA, and, through G, draw GS parallel to ZA; and, through S, draw CH parallel to BZ. The area ASH will be equal to the area SBC.

By the construction, the angle ARB = AHS, and the angle WCH = WBR;

also BR : BI : : rad. : sine SCB,
and BF : BR : : sine AHS : rad.

Joining these ratios, BF : BI : : sine AHS : sine SCB.

By the construction, and the similarity of the triangles BGS, BZA.

$$BF : BI :: BF^2 : BG^2 :: GZ^2 : BG^2 :: SA^2 : SB^2.$$

Wherefore $SA^2 : SB^2$: : sine AHS : sine SCB

By trigonometry, SH : SA : : sine SAH : sine AHS

and SB : SC : : sine SCB : sine SAH = sine SBC

Joining these ratios, SA × SH : SB × SC : : 1 : 1.

Therefore SA × SH = SB × SC.

But the angle ASH being equal to the angle BSC, the area of the triangle ASH will be to the area of the triangle BSC, as SA × SH is to SB × SC; and, since SA × SH is equal to SB × SC, the area of the triangle ASH is equal to the area of the triangle SBC.

The point S having been thus determined, (fig. 9.) if the line CH is drawn through it, inclined to BA at an angle equal to the vessel's inclination from the upright, the water's surface will coincide with the line CH.

To proceed with the construction of this case; bisect BA in D, (fig. 9.) and join WD: let G represent the centre of gravity of the vessel, and E the centre of gravity of the volume displaced, when the vessel floats upright. Let M and I be the centres of gravity of the triangles SAH, SBC; and ML, IK, lines drawn perpendicular to CH, through the points M and I respectively. Through G, draw GU parallel to CH; and, through E, draw EV parallel and equal to KL. In EV, take ET to EV as the area ASH is to the area representing the total volume immersed. Through T, draw TZ perpendicular to GU. GZ will be the measure of the vessel's stability.

As in the preceding cases, let BA be denoted by the letter t, and put the sine of ASH $= s$, sine SAH $= a$, sine SHA $= b$, sine SCB $= c$; the total volume immersed $=$ V.

$$\text{By trigonometry, } SL = \frac{SA}{3} \times \sqrt{4 + \frac{s^2 \times \overline{1-b^2}}{b^2} + \frac{4s \times \sqrt{1-a^2}}{b}}$$

$$SK = \frac{SB}{3} \times \sqrt{4 + \frac{s^2 \times \overline{1-c^2}}{c^2} - \frac{4s \times \sqrt{1-a^2}}{c}}$$

And, since the area ASH $= \frac{SA^2 \times sa}{2b} = \frac{SB^2 \times sa}{2c}$, and V is the area representing the entire volume immersed, the measure of stability, or

$$GZ = \frac{SA^3 \times sa}{6Vb} \times \sqrt{4 + \frac{s^2 \times \overline{1-b^2}}{b^2} + \frac{4s \times \sqrt{1-a^2}}{b}}$$

$$+ \frac{SB^3 \times sa}{6Vc} \times \sqrt{4 + \frac{s^2 \times \overline{1-c^2}}{c^2} - \frac{4s \times \sqrt{1-a^2}}{c}} - ds.$$

In which expression, SA $= \frac{t \times \sqrt{b}}{\sqrt{b} + \sqrt{c}}$, and SB $= \frac{t \times \sqrt{c}}{\sqrt{b} + \sqrt{c}}$.

Let the sides of a vessel be plane surfaces, inclined to each other at an angle of 30°; the vessel's inclination from the upright $= 15°$; $BA = t = 100$; $GE = d = 13$; the angle $SAH = 105°$; $AHS = 60°$: $BCS = 90°$. By calculating the value of the line GZ, according to the solution just given, it is found

$$\text{that } \frac{SA^3 \times sa}{6Vb} \times \sqrt{4 + \frac{s^2 \times \overline{1 - b^2}}{b^2} + \frac{4s \times \sqrt{1 - a^2}}{b}} = 3.1155$$

$$\text{and } \frac{SB^3 \times sa}{6Vc} \times \sqrt{4 + \frac{s^2 \times \overline{1 - c^2}}{c^2} - \frac{4s \times \sqrt{1 - a^2}}{c}} = 3.1075$$

Sum of these values = 6.2230

ds - - - = 3.365

Finally, the measure of the vessel's stability,

$$\text{or } \frac{SA^3 \times sa}{6Vb} \times \sqrt{4 + \frac{s^2 \times \overline{1 - b^2}}{b^2} + \frac{4s \times \sqrt{1 - a^2}}{b}}$$

$$+ \frac{SB^3 \times sa}{6Vc} \times \sqrt{4 + \frac{s^2 \times \overline{1 - c^2}}{c^2} - \frac{4s \times \sqrt{1 - a^2}}{c}} - ds = GZ = 2.858$$

If the weight of the vessel should be 1000 tons, the force of stability will be equivalent to that weight of pressure, acting at the distance of 2.85 from the axis; or the weight of 57.0 tons, acting at the distance of 50 from the axis.

If the sides should be inclined at an angle of 60°, instead of 30°, the measure of stability will be 2.92; and the effort to turn the vessel equal to 1000 tons, acting at the distance 2.92, or 58.4 tons acting at the distance of 50 from the axis.

The sides of vessels are not unfrequently formed so as to coincide with the sides of an isosceles wedge, or are so little curved as to approximate nearly to that figure, at least so far as that portion of the sides extends which may be immersed in, or may emerge from, the water, by the vessel's inclination. The preceding solution being expressed in terms which are rather

complicated, another solution is subjoined, by which the measure of stability is exhibited in more simple terms. The investigation is troublesome; but the conciseness of the result, and the readiness with which it is applied to practical cases, compensate for the difficulty of obtaining it.

Let the isosceles triangle BAF (Tab. X. fig. 11.), represent a vertical section of the vessel; the base of which, BA, coincides with the water's surface, when the vessel floats upright. Bisect BA in D, and join FD. Let G be the centre of gravity of the vessel, and take FE to FD as 2 to 3; E will be the centre of gravity of the immersed volume when the vessel floats upright. Draw the line CH, * intersecting the line BA at an angle ASH, equal to the given angle of the vessel's inclination from the perpendicular, and cutting off the area ASH equal to the area BSC. When the vessel is inclined through the angle ASH, the line of intersection with the water's surface will coincide with CH. Bisect CH in the point N, and join FN: take FQ to FN as 2 to 3. Q is the centre of gravity of the area CFH, representing the volume immersed when the vessel is inclined. Through Q, draw QM perpendicular to CH; and, through G, draw GZ perpendicular to QM. GZ is evidently the measure of the vessel's stability.

To obtain an analytical value of the line GZ, through Q, draw OQP parallel to CH; through G, draw GR parallel to QM; and, through E, draw ET perpendicular to QM. In this investigation it will be expedient, first, to express in general and known terms the line FW; secondly, the line WQ, which is to MW as the sine of the vessel's inclination to radius: this will give the value of MW, which being added to WF before

* By the construction, p. 228.

found, the sum will be the line FM; from which, if FE, or $\frac{2}{3}$ of FD, be subtracted, there will remain the line ME; which is to ET as radius is to the sine of the inclination EMT, or ASH. ET will therefore be expressed in known terms; from which, if ER be subtracted, the remaining line will be RT, or GZ, the measure of the vessel's stability, analytically expressed.

By the construction, the area FBA is equal to the area FCH; and, since the area BAF is to the area IKF in the same * proportion which the area FCH bears to the area FOP, it follows, that the area FIK is equal to the area FOP. Also, because CN is equal to NH, and OP is parallel to CH, it follows, that OQ is equal to QP. For brevity, let the angle KYP, or ASH, be denoted by the letter S; FPO = FHC by P; POF = HCF by O; also let the angle PFO be made = F.

Because the areas IFK, PFO, are equal,

$\frac{FO \times FP \times \text{sine } F}{2} = \overline{FE}^2 \times \text{tang.} \frac{1}{2} F$, radius being $= 1$: wherefore,

$FP = \frac{2\, FE^2 \times \text{tang.} \frac{1}{2} F}{FO \times \text{sine } F} = \frac{FE^2 \times \text{sec.}^2 \frac{1}{2} F}{FO}$; and, because

$FO = \frac{FP \times \text{sine } P}{\text{sine } O}$, by substitution $FP^2 = \frac{FE^2 \times \text{sec.}^2 \frac{1}{2} F \times \text{sine } O}{\text{sine } P}$;

and therefore $FP = FE \times \text{sec.} \frac{1}{2} F \times \sqrt{\frac{\text{sine } O}{\text{sine } P}}$: but sine FWP = cos. S.

Wherefore, FW : FP : : sine P : cos. S; or

$FW : FE \times \text{sec.} \frac{1}{2} F \times \sqrt{\frac{\text{sine } O}{\text{sine } P}}$: sine P : cos. S; consequently,

$$FW = \frac{FE \times \text{sec.} \frac{1}{2} F \times \sqrt{\text{sine } O \times \text{sine } P}}{\text{cos. } S}.$$

By investigation,† founded on the rules of trigonometry, it

* Each of these proportions being as 9 to 4. † See Appendix.

appears that $\sqrt{\text{sine O} \times \text{sine P}} = \frac{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}}{\text{sec.} \frac{1}{2} F \times \text{sec. } S}$:

which quantity being substituted instead of $\sqrt{\text{sine O} \times \text{sine P}}$, in the value of FW just found, the result will be

$$FW = FE \times \sqrt{1 - \text{tang.}^2 \tfrac{1}{2} F \times \text{tang.}^2 S}.$$

It is found also, from trigonometrical rules, that

* $WQ = FE \times \frac{\text{tang.}^2 \frac{1}{2} F \times \text{tang. } S \times \text{sec. } S}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}}$, and since

WQ : WM : : sine S : rad. we have

$WM = \frac{FE \times \text{tang.}^2 \frac{1}{2} F \times \text{tang. } S \times \text{sec. } S}{\text{sine } S \times \sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}}$, or because

$\frac{\text{tang. } S}{\text{sine } S} = \text{sec. } S$, $WM = FE \times \frac{\text{tang.}^2 \frac{1}{2} F \times \text{sec.}^2 S}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}}$; and, since

$FW = FE \times \sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}$, and $FM = WF + WM$,

we obtain the value of $FM = FE \times \frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang}^2 S}}$:

Therefore $ME = FM - FE = FE \times \overline{\frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}} - 1}$,

and $ET = FE \times \text{sine } S \times \overline{\frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}} - 1}$.

This value of ET is inferred from supposing the area BFA to represent the entire volume immersed, and which $= \frac{t^2}{4 \times \text{tang.} \frac{1}{2} F}$ t being equal to the line BA.

If, the sides BC, AH, remaining the same, the figure and magnitude of the immersed volume should be changed, so as to be represented by any other quantity V†, the line ET will be increased or diminished in the inverse proportion of the entire volumes immersed, that is

as $V : \frac{t^2}{4 \times \text{tang.} \frac{1}{2} F} : : FE \times \text{sine } S \times \overline{\frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}} - 1} : ET$.

And, since $FE = \frac{t}{3 \text{ tang.} \frac{1}{2} F}$,

* See Appendix.

† See pages 213 and 214.

$$ET = \frac{t^3 \times \text{sine S}}{12V \times \text{tang.}^2 \frac{1}{2} F} \times \overline{\frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}} - 1};$$

and the measure of the vessel's stability, expressed in general and known terms, will be

$$* GZ = \frac{t^3 \times \text{sine S}}{12V \times \text{tang.}^2 \frac{1}{2} F} \times \overline{\frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}} - 1} - d \times \text{sin. S}.$$

When the angle of inclination S is evanescent, or in a practical sense very small, the expression becomes

$GZ = \frac{t^3 \times \text{sine S}}{12V} - d \times \text{sine S}$, agreeing with the solution given by M. EULER † in this particular case.

If the inclination of the sides BF, AF, should be evanescent, the sides will become parallel to each other, and to the masts, both above and beneath the water-line; a case which has already been solved ‡: and consequently, the solution of case 1. ought to agree with that which has been just given for the stability, when the two sides are inclined at a given angle, assuming that angle as evanescent. Assuming, therefore, the angle BFA evanescent, and S of any finite magnitude in the general value of GZ, above determined, we have

$\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S} = \frac{2 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}{2}$, and

$$\frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}} = \frac{2 + 2 \times \text{tang.}^2 \frac{1}{2} F + \text{tang.}^2 \frac{1}{2} F \times \text{sec.}^2 \frac{1}{2} F \times \text{tang.}^2 S}{2};$$

and therefore $\S GZ = \frac{t^3 \times \text{sine S}}{12V \times \text{tang.}^2 \frac{1}{2} F} \times \frac{\text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S + 2 \times \text{tang.}^2 \frac{1}{2} F}{2}$ $- d \times \text{sin. S}$.

* This expression for the measure of stability, is evidently more simple, and better adapted to practical application, than that which is inserted in page 229. The present result might perhaps be obtained by more concise methods: the investigation here given is the best that occurred to the author, after repeatedly endeavouring to discover some other, requiring fewer trigonometrical calculations.

† Theory of the Construction and Properties of Vessels, chap. viii.

‡ Case 1.

$$\S\ ET = \frac{t^3 \times \text{sin. S}}{12V \times \text{tang.}^2 \frac{1}{2} F} \times \overline{\frac{2 + 2 \times \text{tang.}^2 \frac{1}{2} F + \text{tang.}^2 \frac{1}{2} F \times \text{sec.}^2 \frac{1}{2} F \times \text{tang.}^2 S}{2} - 1},$$

or because sec. $\frac{1}{2} F = 1$, $ET = \frac{t^3 \times \text{sin. S}}{12V} \times \frac{2 + \text{tang}^2 S}{2}$.

$$\text{or } GZ = \frac{t^3 \times \text{sine } S}{24V} \times \overline{\text{tang.}^2 S + 2} - d \times \text{sin. } S,$$

$$\text{or } GZ = \frac{t^3 \times \text{tang. } S}{24V} \times \overline{\text{cos. } S + \text{sec. } S} - d \times \text{sin. } S,$$

which is the measure of stability, when the inclined sides AF, BF, become parallel, the angle F vanishing. But this quantity is the measure of stability when the sides are parallel, as determined by direct investigation*; by which agreement the consistency of the two solutions is evinced.

To exemplify the general solution for the case of the sides inclined at a given angle, suppose the angle BFA to be $30° =$ F, let $S = 15°$, $AB = t = 100$, $GE = d = 13$, $V = 3600$.

From the analytical value of the line GZ, we obtain

$$\frac{t^3 \times \text{sine } S}{12V \times \text{tang.}^2 \frac{1}{2} F} \quad - \quad - \quad - \quad = 83.4464$$

$$\frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}} - 1 \quad - \quad - \quad = .07457$$

$$83.4464 \times .07457 \quad - \quad = 6.223$$

$$d \times \text{sin. } S \quad - \quad - \quad = \underline{3.365}$$

$$\text{and GZ, the measure of stability} \quad = 2.858,$$

precisely agreeing with the result calculated by the solution, in pages 229 and 230, which has no apparent similitude or relation to the value for stability, as expressed according to this last investigation, which is

$$GZ = \frac{t^3 \times \text{sin. } S}{12V \times \text{tang.}^2 \frac{1}{2} F} \times \overline{\frac{\text{sec.}^2 \frac{1}{2} F}{\sqrt{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}} - 1} - d \times \text{sin. } S.$$

According to the solution in page 229, the measure of stability is

$$GZ = \frac{SA^3 \times sa}{6Vb} \times \sqrt{4 + \frac{s^2 \times \overline{1 - b^2}}{b^2} + \frac{4s \times \sqrt{1 - a^2}}{b}}$$

$$+ \frac{SB^3 \times sa}{6Vc} \times \sqrt{4 + \frac{s^2 \times \overline{1 - c^2}}{c^2} - \frac{4s \times \sqrt{1 - a^2}}{c}} - ds,$$

* Case 1.

in which value $s = \sin.\ S$; $a = \sin.\ SAH$; $c = \sin.\ SCB$; $SA = \frac{t \times \sqrt{b}}{\sqrt{b} + \sqrt{c}}$, and $SB = \frac{t \times \sqrt{c}}{\sqrt{b} + \sqrt{c}}$. It might not, perhaps, be easy to deduce either of these values from the other, or to demonstrate their equality, otherwise than by the separate investigations from which they have been inferred; and yet these quantities are not approximations to equality, but are strictly and mathematically equal.

CASE VII.

The sides of a vessel are coincident with the sides of a wedge, meeting, if produced, at an angle which is above the water's surface.

The sides of a vessel are represented by the lines qb, cd, (fig. 12.) inclined at an angle, so as, if produced, to meet at the point w above the water's surface, which is coincident with ba; the lines wa, wb, are assumed equal. Suppose the vessel to be inclined from the perpendicular through any given angle; let a line cb be drawn, intersecting the line ba at the given angle of inclination, and cutting* off the area asb equal to the area bsc: when the vessel is inclined to the given angle from the upright, the water's surface will be coincident with the line cb. Let m and i represent centres of gravity of the areas asb, bsc, respectively, and let the line kl be constructed as in the former cases. Let g be the centre of gravity of the vessel, situated in the line we, which is drawn perpendicular to and bisects ba, and let e be the centre of gravity of the volume displaced; making ev parallel and equal to lk, take et to ev as the area bsc is to the area representing the entire volume immersed. Through g, draw gu parallel to cb, and, through t, draw tz perpendicular to gu. gz will be the measure of the vessel's stability.

* Page 228.

From this construction, the following proposition is to be inferred.

The sides of a vessel are plane surfaces, represented, (fig. 9.) when produced, by the equal lines AW, BW, which meet in the point W, beneath the water-line. The sides of another vessel (fig. 12.) are also plane surfaces inclined to each other at the same angle as in the former case, and represented by the equal lines *aw*, *bw*, which meet at the point *w* above the water-line: suppose the breadth of both vessels to be equal at the water-line, and the angle BWA = the angle *bwa*; if the distances between the centres of gravity of the vessels and of the immersed volumes are equal, and the weights of the vessels are also equal, the proposition affirms, that the stabilities of the two vessels, when inclined to the same angle from the upright, will always be equal.

Since the line BA = *b a*, and the angle BAW = the angle *baw*, (fig. 9 and 12.) by the conditions of the proposition, if the angle BAW be applied over the angle *baw*, the point A coinciding with the point *a*, it follows, that the point W, and the point B, must coincide with the point *w* and the point *b* respectively; and, since the lines BA, *ba*, are divided in the points S, *s*, on the same conditions, namely, so that the lines CH, *c b*, shall be inclined to BA, and *b a*, at the same angle, and shall cut off the areas ASH, *a s b*, equal respectively to the areas BSC, *b s c*; it must follow, that when the line AB is applied so as to coincide with the line *a b*, the point S will coincide with the point *s*; and the angle ASH being equal to the angle *a s b*, by the supposition, the line SH will be equal to the line *s b*; and the triangle ASH will be equal and similar to the triangle *a s b*. The centres of gravity of these triangles, therefore, or the points M and *m*, will coincide, as will also the lines ML, *m l*, which

are drawn through these points perpendicular to CH and *cb*. The line SL will therefore coincide with the line *sl*, and is equal to it. In the same manner, it is proved that the line SK is equal to the line *sk*; consequently, KL is equal to *kl*. And since, by construction, the area ASH is equal to the area BSC, and the area *asb* equal to the area *bsc*; and, on application of the figure AWB to the figure *awb*, the triangle ASH coincides with the triangle *asb*, it follows, that the four areas ASH, *asb*, BSC, *bsc*, are all equal.

But ET* $= \frac{\text{KL} \times \text{area ASH}}{\text{total volume immersed}}$, and $e\,t$† $= \frac{k\,l \times \text{volume } a\,s\,b}{\text{total volume immersed}}$; and, since KL $= k\,l$, and the volume ASH $=$ the volume $a\,s\,b$, KL $\times$ volume ASH $= kl \times$ volume asb; and the entire volume immersed being the same in both vessels, by the supposition, it follows that ET $= e\,t$.

This equality between the lines ET, *et*, is independent of the position of the centres of gravity of the vessels, G, *g*, and also of the position of the centres of gravity, E, *e*, in the lines WD, *wd*. If the distances of GE, *ge*, should be equal, since the angles of inclination from the upright, or EGR, *egr*, are equal by the supposition, it follows that the sines of those angles to equal radii must be equal, or ER $= er$. Subtracting, therefore, ER from ET, and *er* from *et*, the remaining lines RT, *rt*, must be equal, or GZ $= gz$. The stability, therefore, of a vessel, the sides of which are inclined to an angle under the water's surface, is equal to the stability of the vessel of which the sides are inclined to an angle which is above the water's surface: the breadth at the water-line, and the other conditions, being the same in both vessels.

This proposition is not confined to the case here demon-

* Fig. 9. See page 212. † Fig. 12. See page 212.

strated, being equally true, whatever figure be given to the sides; and whether they are plane or curved, provided the sides under the water-line in one vessel are similar and equal, and similarly disposed, in respect of the water-line, to the sides of the other vessel above the water-line. QC, HO, (fig. 13.) represent the sides of a vessel projecting outward above the water-line, and inclined inward under the water-line. Suppose the vessel to be inclined from the upright through any given angle, and let CH be supposed drawn inclined to the line BA at the given angle, and cutting off the area ASH equal to the area SBC: when the vessel is inclined, the water's surface will coincide with the line CH.

Let the sides QC, OH, be conceived to revolve round the line BA as an axis, through 180°; the position of the sides will be reversed, as represented in fig. 14: the sides which projected outward above the water-line (fig. 13.) equally project outward under the water-line in fig. 14. and are similarly situated in respect to the water-lines BA, *ba*. In like manner, the sides which are inclined inward under the water-line, in fig. 13. are equally inclined inward above the water-line in fig. 14.; and are also similarly situated in respect to that line. If M, I, are the centres of gravity of the areas ASH, BSC, and *m*, *i*, the centres of gravity of the areas *ash*, *bsc*, as in the former cases, and perpendicular lines be drawn through them, ML, IK, and *ml*, *ik*; by arguments similar to those which were used to demonstrate the preceding proposition, it will be evident that the lines KL, *kl*, are equal; also that the areas ASH, BSC, *ash*, *bsc*, are all equal: and, by proceeding to construct the measures of stability corresponding to the two cases, it will appear that $GZ = gz$; the weight of both vessels, and consequently the entire volumes immersed under water, being the same. The conclusion is, that, the

other conditions remaining the same, if the position of the sides should be reversed, in the manner described in the proposition, the stability, at equal angles of inclination, will remain the same.

It may be proper in this place to remark, that the metacentric curve, described by M. BOUGUER,* and M. CLAIRBOIS,† and applied to the preceding cases, does not appear to have any relation to the stability of vessels, except in the single point where the curve intersects the vertical axis; and therefore can be applicable only in the case when the angle of the vessel's inclination from the upright is evanescent. Let FBC, DAH, (fig. 15.) represent the sides of a vessel, BA coinciding with the water's surface when the vessel floats upright: bisect BA in S, and draw ISE perpendicular to BA. Let E be the centre of gravity of the volume immersed. Suppose the vessel to be inclined through a very small angle AS *a*, so that the water's surface shall now coincide with the line *ba*; and let the centre of gravity of the volume immersed be Q. Through Q, draw the line QW*z* perpendicular to *ba*, intersecting the line IE in the point W. This point is called by M. BOUGUER the metacentre. One of the principal properties of this point is, that whenever the centre of gravity of the vessel is situated beneath it, any where in the line WE, (suppose at G,) the vessel will float permanently, with the line IE vertical; but that, if the centre of gravity is placed above the metacentre, suppose at *g*, the vessel will overset, from that position; for, drawing GZ, *gz*, perpendicular to Q*z*, if the vessel should be inclined through a small angle AS *a*, so as to immerse the portion of the side A*a*, the force of pressure acting in the direction of the line QZ, to turn the vessel round an axis passing horizontally through G, will elevate the parts adjacent to A, so as to restore the upright position: whereas, if the centre of gravity

* *Traité du Navire, p.* 270.

† CLAIRBOIS, p. 289, *et seq.*

should be placed above the metacentre, suppose at *g*, the same force of the fluid's pressure, by turning the vessel round an axis passing through *g*, must immerse further the portion of the side A*a*; and this immersion, being continued, will cause the vessel to overset. Another property of this point has been demonstrated by M. EULER,* and other authors; which is, that when the angles of a vessel's inclination are evanescent, or very small, the effect of stability, to restore the vessel to the upright position, will be as the sine of the angle of inclination GWZ † and the line WG jointly: at the same small angles of inclination, the stability of different vessels will be in proportion to the line WG, or distances of the metacentre above the centre of gravity.

Let the curve EQ*q* (Tab. XI. fig. 16.) represent the line traced by the successive centres of gravity of the immersed volumes, while the vessel is inclined from the upright through any angle ASH. M. BOUGUER demonstrates, that a tangent to this curve in any point Q, will be parallel to the water's surface CH, corresponding to that point: if, therefore, through any two adjacent points Q and *q*, in the curve EQ*q*, lines QM, *q*N, are drawn perpendicular to the lines CH, *ch*, respectively, the intersection of those lines in the point X will be the centre of curvature, and XQ, X*q*, will be the radii of a circle, which has the same curvature with the curve EQ in the point Q. For the same reasons, the line WE (fig. 15, 16.) is the radius of a circle which has the same curvature with the curve EQ in the point E. The point W has been denominated the metacentre corresponding to the upright position of the vessel, when the line WGE is perpendicular to the water's surface. M. BOUGUER denomi-

* Theory of the Construction of Vessels, chap. 8. book i.

† To radius $= 1$.

nates the point X the metacentre corresponding to the position when the vessel has been inclined from the upright through the angle ASH; and the curve WX is termed the metacentric curve, being the line traced by the successive metacentres, or intersections, of the lines QM, q N, drawn perpendicular to the lines in which the vessel is intersected by the water's surface, while it is gradually inclined. Consequently, according to this construction, the metacentric curve WX is the evolute, of which the curve EQq is the involute.

The construction and properties of the metacentric curve being a subject of geometrical reasoning, considered purely as such, are liable neither to ambiguity nor error; but, on what grounds these properties are applied to measure the stability of vessels, or to estimate their security from oversetting, when much inclined from the upright, is not explained by M. BOUGUER, M. CLAIRBOIS, or any other author I have had an opportunity of consulting: yet the opinions expressed by these authors on the subject in question, have been adopted by many persons as established principles; and, being of some importance in the practice, as well as theory, of naval architecture, it cannot be thought superfluous to pay some farther attention to them.

M. BOUGUER,* having demonstrated the property of the metacentre, which gives security from spontaneously oversetting, to a vessel, whenever the centre of gravity is situated beneath it, proceeds to observe, that his theorem, being founded on supposing the angles of the vessel's inclination as evanescent, or extremely small, such as a vessel may experience in smooth water, cannot be relied on for ascertaining the safety of ships,

* *Traité du Navire*, p. 269.

when agitated by the winds and waves in open sea, where the inclinations from the upright must often become considerable. In order to extend the application of his theorem to the larger angles of inclination, he proposes to examine whether the metacentre ascends or descends as the vessel is gradually inclined.* To effect this, the curve line EQ*q* (fig. 16.) is to be traced, by finding the successive centres of gravity of the volumes immersed while the vessel is inclined; and, from this curve the metacentric curve WX is to be defined: the point where the metacentric curve meets the vertical axis in W, is the metacentre corresponding to the position when the vessel floats upright and quiescent. He observes, that if the metacentre X ascends from its original position W, while the vessel is inclined gradually from the perpendicular, the vessel will be secure from oversetting; but will be insecure, if that point should descend while the vessel is inclined. No demonstration of this proposition is given, either by M. BOUGUER, or by M. CLAIRBOIS, who undertakes to explain the principles delivered in this chapter of M. BOUGUER's work.† If the proposition has been suggested by some analogies which subsist between the construction of the lines EW, QX, and other lines similarly drawn, they will be insufficient to establish the truth of it. The analogies are such as the following. W being the metacentre, and E the centre of gravity of the volume displaced, when the vessel floats upright, WE is the radius of curvature to the curve EQ*q*, at the point E; X being also the metacentre, constructed according to the method which has been described, when the vessel has been inclined through an angle ASH, and Q the centre of gravity of the corresponding volume immersed; XQ is the

* *Traité du Navire*, p. 271. † CLAIRBOIS *sur l'Architecture Navale*, p. 289, *et seq.*

radius of curvature of the curve EQ at the point Q. Also, EW is perpendicular to the water's surface AB, when the vessel floats upright; and XQ is perpendicular to the water's surface, when the vessel is inclined through the angle ASH. When the vessel floats upright, the stability is measured by the sine of inclination and the line GW jointly; and therefore the angle of inclination being given, will be measured by the line GW, and will depend in some ratio or proportion on the line EW, when GE remains the same, or when G is made to coincide with E.

The question is, whether the stability, when the vessel is inclined to the angle ASH, will depend in a similar degree on the line QX? Respecting the supposed analogy it may be remarked, that one condition absolutely necessary to establish it is wanting; namely, the centre of gravity G ought to be situated in the line XQ; but it is considerably distant from that line, being placed in the vertical axis of the vessel WGE. This material difference in the conditions corresponding to the two cases, is sufficient to destroy all inference from analogy, even if arguments of this kind could be admitted, in geometrical subjects, to supply the place of demonstration. It is not difficult to shew geometrically, in what position and circumstances of the vessel the line XQ will be the correct measure of its stability. Suppose that, by any alteration in the distribution of the ballast or lading, the centre of gravity should be removed from the line WGE to the line XGQ, the vessel will float permanently with the line XQ perpendicular to the horizon, and the mast WE will be inclined to it at the angle = ASH. Since XQ is the radius of curvature of the curve EQ at the point Q, and is also perpendicular to CH, the point X will be the true position of the metacentre, corresponding to the float-

ing position of the vessel, when the centre of gravity is situated out of the vertical axis in the line XQ, and Q is the centre of gravity of the volume displaced. The measure of stability, when the inclination is any small angle, will be the sine of that angle and the line XG jointly; comparing, therefore, the stability of the vessel when the centre of gravity is situated in the line WGE, with the stability when the centre of gravity is in the line XGQ, the proportion of the two stabilities, at equal small angles of inclination, will be as the line WG is to the line XG; if the centre of gravity G should coincide with the point E in the first case, and with the point Q in the latter case, a condition often adopted by M. Bouguer, the stabilities will be in the proportion of the lines WE to XQ, or in a triplicate ratio of the lines BA, CH.

Such is the result of the examination proposed, from which the only inference is, that while the centre of gravity remains situated in the vertical axis WE, (the position it occupies in vessels of every description,) the line XQ cannot be assumed to measure or estimate the stability and security of a vessel at sea, when inclined to the larger angles from the upright. M. Clairbois, to illustrate the principles of M. Bouguer, adopts two instances, which are the same with Case vi. (fig. 9.) and Case vii. (fig. 12.) in these pages. In the former case, the sides coincide with those of an isosceles wedge; the breadth BA at the water-line being the base, and the angle BWA situated under the water's surface. As the vessel thus formed is gradually inclined from the perpendicular, he shews,* that the curve traced by the centre of gravity of the successive volumes immersed is an hyperbola. Of this curve he calculates the successive radii

* Clairbois, p. 291: 295.

of curvature, which he demonstrates to increase continually with the inclination of the vessel: he shews, that the centres of curvature thus found, or successive metacentres, according to M. BOUGUER'S construction, ascend as the vessel is inclined; a circumstance which, according to his principle, imparts security from oversetting. On the contrary, in the other instance, when the sides of a vessel are inclined to an angle which is above the water's surface, (fig. 12.) from a similar mode of reasoning he concludes, that the metacentre descends as the vessel is more and more inclined; which, according to his proposition, would endanger the safety of the vessel, when inclined to considerable angles.

This determination is evidently inconsistent with the solutions of Case VI. and VII. preceding, by which it appears, that the stability acting to restore vessels thus constructed to the upright position, under the conditions that have been stated, will be precisely the same at all equal inclinations from the upright, whether the sides are inclined at an angle beneath or above the water-line; all the other conditions being the same in both cases.

The solution of these questions being connected with a principle of some consequence in the practice of naval architecture, the preceding observations have been offered with a view of stating distinctly the opinions which are contradictory to the solutions of Case VI. and VII. referring to the authors who have treated on the subject, in order that a judgment may be formed by persons conversant in naval architecture, whether the propositions advanced by M. BOUGUER and M. CLAIRBOIS, or the solutions of Case VI. and Case VII. here given, may be relied on, as founded on the genuine principles of geometry and mechanics; for error must exist on one side or the other.

But, until the demonstrations of the Cases VI. and VII. are shewn to be erroneous, and reasons are produced in support of M. BOUGUER's propositions, which he has delivered without any demonstration, it may be allowable to suppose that his opinions are, in these particular instances, ill founded.

The same principles are extended by M. BOUGUER * to express a general value of the distance between the metacentre, and the centre of the immersed part of the ship, when inclined to any angle: this distance he affirms to be $\frac{\text{Flu.}\, \dot{x} \times \overline{y^3 + v^3}}{3p}$; † in which expression y and v are the parts of the total ordinate of the water-section, (when the vessel is inclined,) at the distance x, measured on the longer axis from the initial point; the proportion of y and v being determined by a line drawn parallel to the axis through the centre of gravity of the section; and p is put for the volume immersed.

When the centre of gravity is situated in the line QX, (fig. 16.) and the angle of inclination very small, the point of intersection of the lines CH, *c h*, will bisect the ordinate CH: in this case the vessel floats permanently with the line QX vertical, and consequently with the line WE, or plane of the masts, inclined to the horizon at the angle ASH. But the line QX, consistently with the preceding observations, cannot be applied to measure the stability or security from oversetting of a ship, when the centre of gravity is placed in the line WE; that is, in the plane of masts which divides the vessel into two parts perfectly similar and equal; the only situation which the centre of gravity can occupy, according to any mode of construction hitherto practised.

* *Traité du Navire*, p. 273.

† No demonstration is given by M. BOUGUER of this proposition.

A few remarks may be added in this place concerning a theorem delivered by M. BOUGUER,* for measuring the stability of vessels when inclined to evanescent angles from the upright. The theorem is this: "When the lengths of vessels "are the same, the stabilities are as the cubes of the breadths." This theorem seems at first view to stand independent of, and not to require, any subsequent explanation: the author immediately applies it to the discussion of some points respecting the stability of vessels. If any person, relying on the author for the truth of this theorem, should only pay attention to the proposition as it is here expressed, he would entertain an opinion on the subject of stability which is altogether erroneous. M. BOUGUER,† in a subsequent page, gives a satisfactory account of the limitations and restrictions under which the theorem in question is to be understood. He observes, that a restriction ought to be applied to the conditions of this proposition, in order to insure the exact correctness of it; which is, that the whole weight of the vessel shall be concentered in the centre of gravity of the displaced volume; a condition which may be deemed amongst the most extreme cases that can be devised, and such as is rarely known to exist.‡ The vessel's centre of gravity not being supposed coincident with the centre of the displaced volume; M. BOUGUER § gives the true measure of stability when the angles of inclination are

* *Traité du Navire*, p. 299. † Ibid. p. 299 and 300.

‡ In vessels of burden, the freights of which consist principally of iron, or other metallic bodies, or blocks of stone, the vessel's centre of gravity may be so depressed as to coincide with, or even to be situated under, the centre of the immersed volume. But such a disposition causes many inconveniences in the ship's sailing; and is never adopted when it is possible to raise the centre of gravity to a higher position.

§ *Traité du Navire*, p. 300.

evanescent; the only objection to which is, that it stands in the author's page as being explanatory, and illustrative of a proposition before delivered: whereas, it is in fact the real proposition for measuring the horizontal stability of vessels; the proposition it is intended to explain being a particular case of it, and requiring a condition which scarcely ever takes place in the practice of constructing and adjusting ships for sea.

CASE VIII.

The sides of a vessel are parallel to the masts above the water-line, (fig. 17.) and project outward beneath it.

In the second Case, (fig. 4.) the sides project outward above the water-line, and are parallel to the masts under it. In Case VIII. the disposition and form of the sides are the reverse of the form according to Case II. If, therefore, the angle of projection of the sides under the water, according to Case VIII. should be equal to the angle at which the sides project above the water, according to Case II. the other conditions being the same, the stabilities* of the two vessels will be equal, at all equal inclinations from the quiescent position. The solution of this case must of consequence be precisely the same with the solution of CASE II. and need not be here repeated.

CASE IX.

The sides of a vessel are parallel to the masts above the water-line, (fig. 18.) and are inclined inward beneath it.

In this case, the position of the sides is the reverse of that which is described in Case III. (fig. 6.) If, therefore, the angles at which the sides are inclined inward, according to Case IX.

* Proposition subjoined to Case VII. page 237.

under the water-line, should be equal to the angle at which the sides are inclined inward above the water-line, according to Case III. all the other conditions being the same, the stabilities of the two vessels will be equal, at all equal inclinations from the upright. The solution of Case IX. is therefore to be derived from that of Case III.

CASE X.

The sides of a vessel coincide with the surface of a cylinder, the vertical sections being equal circles.

Let QBOAH (fig. 19.) represent a vertical section of the vessel. The surface of the water coincides with the line BA, when the vessel floats upright. Suppose the vessel to be inclined from the quiescent position, through an angle ASH, so that the water's surface shall intersect the vessel's, when inclined, in the line CH. Bisect the line BA in D, and the line CH in Y; and, through the points D and Y, draw OD, FY, perpendicular to the lines BA, CH, respectively, and meeting, when produced, in the point M, which is the centre of the circle. The angle ASH is the inclination of the vessel from the perpendicular; and, being the inclination of the lines BA, CH, which are perpendicular to the lines OM, FM, respectively, the inclination of the lines OM, FM, or the angle DMF, will be equal to the angle ASH. Let E be the centre of gravity of the area BOA, representing the volume displaced, when the vessel floats upright, and quiescent. In the line MF, take MQ equal to ME; Q will be the centre of gravity of the area CFH, representing the volume displaced when the vessel is inclined. Let G be the centre of gravity of the vessel; and, through E, draw ET perpendicular to MF; and, through G, draw GZ perpendicular to MF, intersecting that line in the

point Z: GZ is the measure of the vessel's stability. For, since Q is the centre of gravity of the volume immersed, when the vessel is inclined, and the line MF is drawn through it, perpendicular to the water's surface CH, QM will be the direction in which the pressure of the fluid acts, to turn the vessel round an axis passing through G; and GZ, being the perpendicular distance of this line from the centre of gravity, will be the measure of the vessel's stability.

Let the sine of the angle of the vessel's inclination ASH, or OMF, be represented by the letter s to radius $= 1$: by the properties of the circle ME $= \frac{2DA^3}{3 \times \text{area BOA}} = \frac{BA^3}{12 \times \text{area BOA}}$; if, therefore, BA be made $= t$, ME $= \frac{t^3}{12 \times \text{area BOA}}$; and ET $= \frac{t^3 s}{12 \times \text{area BOA}}$.

The area representing the volume displaced is here considered as entirely circular: but if it should be of that form only to the extent of the sides *AH, BC, the remaining part of the area being of any other figure, and the whole area under water should be denoted by V, the line ET will be $= \frac{t^3 s}{12 \times \text{area BOA}} \times \frac{\text{area BOA}}{V}$, or ET $= \frac{t^3 s}{12V}$. Let GE be denoted by d; then ER $= ds$, and RT, or the measure of the vessel's stability †GZ $= \frac{t^3 s}{12V} - ds$.

* Proposition and observations in pages 213, 214.

† In this expression for the measure of stability, s is the sine of the angle of the vessel's inclination, whatever be its magnitude: this value, for the stability of vessels which have a circular form, is the same with that which M. Bouguer gives for vessels of any form, when the angles of inclination are evanescent, the breadths at the water-line being $= t$, and the other conditions the same; from which circumstance, the following remarkable conclusion is inferred: if the measure of stability should be calculated for finite angles of inclination, by the rule M. Bouguer has given for the

Let $t = 100$, $s =$ sin. $15°$ to rad. $= 1$. $d = 13$, $V = 3600$. According to these conditions, $GZ = 2.63$. If, therefore, the vessel's weight should be 1000 tons, the stability will be equivalent to the weight of 1000 tons, acting to turn the vessel at the distance of 2.63 from the axis passing through G, or equivalent to a weight of 52.5 tons, acting at a distance of 50 from the axis.

CASE XI.

The vertical sections of a vessel are terminated by the arcs of a conic parabola.

Let the parabola BLA (fig. 20.) represent a vertical section of a vessel, floating with the axis DL perpendicular to BA, which coincides with the water's surface. G is the centre of gravity of the vessel. Suppose a ship, so formed, to be inclined from the upright through a given angle MOI. The breadth BA, and depth from the water-line, DL, being given, it is required to construct the measure of the vessel's stability.

The principal parameter being given from the conditions of the construction, from the vertex L set off LF, equal to a fourth part of the parameter: F is the focus of the parabola. In the line LF, take LI to LF, as the tangent of the given angle MOI to radius; and, in the line LI, take LX to LI, in the same proportion of the tangent of the angle MOI to radius. Through the point X, draw XV perpendicular to XL, intersecting the curve in the point V; set off LN equal to XL: join NV, which produce indefinitely, in the direction NVW;

angles of inclination that are evanescent, the stability of all vessels, at equal inclinations, thus calculated, whatever be their forms, would be the same as if the vertical sections were circular; the breadths at the water-line, position of the centres of gravity, and other elements, being the same.

NW is a tangent to the curve in the point V: through the point V, draw VK parallel and equal to DL; and, through the point K, draw CH parallel to NW: let DL be divided into five equal parts, and let LE be taken equal to three of those parts: make VQ equal to LE; and through Q draw $P\Gamma$ perpendicular to NW; through G draw GZ perpendicular to ΓP: GZ is the measure of the vessel's stability, when inclined from the upright through the given angle MOI. The demonstration follows. Through E, draw ET perpendicular to ΓP; and, through G, draw GR parallel to ΓP; let the parameter of the curve be denoted by p.

By the construction, LX : LI : : LI : LF : : tang. MOI to rad.
therefore - LX : LF : : tang.^{2} MOI : rad.^{2} and
and - LX : 4LF : : tang.^{2} MOI : 4rad.^{2}

By the properties of the curve,

LX : XV : : XV : 4LF
wherefore - LX : 4LF : : LX^2 : XV^2.
But - - LX : 4LF : : tang.^{2} MOI : 4rad.^{2}
therefore - LX^2: XV^2: : tang.^{2} MOI : 4rad.^{2}
and - - LX : XV : : tang. MOI : 2 rad.

or, since LX $= \frac{1}{2}$XN

$\frac{1}{2}$XN : XV : : tang. MOI : 2 rad.
or - - XN : XV : : tang. MOI : rad. but, by
the construction, XN : XV : : tang. XVN : rad.

consequently tang. XVN is equal to the tangent of MOI to the same radius; and therefore the angle XVN is equal to the angle MOI, or the given angle of the vessel's inclination from the upright. Moreover, since it appears from the construction, that the angle XVN is equal to the angle $N\Gamma P$, $N\Gamma P$ is equal to the vessel's inclination from the upright, and because the

line BA is parallel to XV, and the line CH parallel to NW, by the construction, it follows, that the angle ASH is equal to the angle XVN; wherefore the angle ASH is also equal to the angle MOI, or the given angle of inclination from the upright. VK being parallel to DL, and therefore a diameter of the curve to the point V, and CH being drawn parallel to NVW, which is a tangent to the curve in the point V, it follows, that VK bisects the line CH in the point K; KH therefore will be an ordinate to the diameter VK: and, since VK is by construction equal to DL, and DL, VK, are abscissæ of the segments BLA, CVH, respectively, it is known, from the properties of the figure, that the area of the segment BLA is equal to the area of the segment CVH; and consequently the area of the figure ASH will be equal to the area of the figure BSC. And since, when the vessel floats upright, the line AB coincides with the water's surface, and the area of the segment ALB is equal to the area of the segment CVH, it follows, that when the vessel is inclined from the perpendicular, through an angle ASH, equal to the given angle MOI, the surface of the water will intersect the vessel in the line CH. Moreover, since LE is to LD as 3 to 5, by the construction, and VQ is to VK in the same proportion of 3 to 5, by the properties of the figure, E is the centre of gravity of the area BLA, and Q is the centre of gravity of the area CVH, which represents the total volume displaced, when the vessel is inclined through an angle ASH, or MOI; and the line ΓQP being, by construction, drawn perpendicular to the water's surface CH, will be a vertical line passing through the centre of gravity Q of the volume displaced CVH: and GZ, drawn through the centre of gravity G, perpendicular to this line, will be the measure of

the vessel's stability, when inclined from the perpendicular through the given angle MOI.

From the preceding construction and demonstration, a property of stability is inferred, which may be expressed in the following proposition.

If the vertical sections of a vessel are terminated by the arcs of a conic parabola, and the sides of another vessel are parallel to the plane of the masts, both above and beneath the water-line, the stabilities of the two vessels will be equal at all equal inclinations from the upright, if the breadths at the water-line BA, and all the other conditions, are the same in both cases.

It is thus demonstrated:

For brevity, let the angle of inclination from the upright, or the angle ASH, be denoted by the letter S; let $BA = t$, and $LD = a$: rad. $= 1$.

From the preceding construction and demonstration, it appears that $XV : XN :: 1 : \text{tang. } S$, and by the properties of the figure $\frac{1}{2}XN : XV :: XV : p$, joining these ratios - $1 : 2 :: XV : p \times \text{tang. } S$.

Wherefore $XV = \frac{p \times \text{tang. } S}{2}$,

and $XL = \frac{XV^2}{p} = \frac{p \times \text{tang.}^2 S}{4} = LN$;

also, since $XV : NV :: \cos. S : 1$,

$$NV = \frac{XV}{\cos. S} = \frac{p \times \text{tang. } S}{2 \times \cos. S};$$

and because $LD = VK = a$, and $LE = VQ = \frac{3a}{5}$,

and the angle $VQP = ASH = MOI$, it follows that

$VP = \frac{3a \times \sin. S}{5}$; and therefore

$$NP = NV + VP = \frac{p \times \text{tang. } S}{2 \times \cos. S} + \frac{3a \times \sin. S}{5};$$

therefore, $\Gamma N = \frac{NP}{\sin. S} = \frac{p}{2 \times \cos.^2 S} + \frac{3a}{5} = \frac{p \times \sec.^2 S}{2} + \frac{3a}{5}$,

and $L\Gamma = \Gamma N - LN = \frac{p \times \sec.^2 S}{2} + \frac{3a}{5} - \frac{p \times \text{tang.}^2 S}{4}$;

and, since $LE = \frac{3a}{5}$,

$$\Gamma E = \frac{p \times \sec.^2 S}{2} - \frac{p \times \text{tang.}^2 S}{4},$$

or $\Gamma E = \frac{p}{4} \times \overline{\sec.^2 S + 1}$, and, since the angle $E\Gamma T = S$, $ET = \frac{p \times \sin. S}{4} \times \overline{\sec.^2 S + 1}$,

or $ET = \frac{p \times \text{tang. } S}{4} \times \overline{\cos. S + \sec. S}$.

This is the value of the line ET, when the area representing the volume immersed is terminated throughout by the parabolic arc, the said area being $= \frac{t^3}{6p}$ or $\frac{2}{3} \times BA \times DL$; but, if that form should extend to the sides AH, BC, only, the remaining part of the volume immersed being of any other figure,* and this entire volume should be of any magnitude V, the value of ET corresponding will be $\frac{p \times \text{tang. } S}{4} \times \frac{t^3}{6pV} \times \overline{\cos. S + \sec. S}$, or $ET = \frac{t^3 \times \text{tang. } S}{24V} \times \overline{\cos. S + \sec. S}$. And, since $ER = d \times \sin. S$, TR, or the measure of the vessel's stability, $GZ = \frac{t^3 \times \text{tang. } S}{24V} \times \overline{\cos. S + \sec. S} - d \times \sin. S$; precisely the same quantity which measures the stability at the angle of inclination S,† when the sides are parallel to the masts above and beneath the water-line: a coincidence not a little remarkable, and such as would not probably have been supposed to exist, except from the evidence of demonstration.

From this proposition it is inferred, that if the sides of a

* See pages 213, 214.

† Case 1. page 216.

vessel coincide with the arcs of the conic parabola, and the sides of another vessel coincide with the arcs of another conic parabola, whatever be the form thereof, varying according to the parameter, the weights of the vessels, breadths at the water-line, and the other conditions being the same in both cases, the stabilities of the two vessels, at all equal angles of inclination, will be equal. If, for instance, the forms of two vessels should be such as are represented in Tab. XII. fig. 21. and fig. 22. the weights and other conditions being the same, the stabilities of each of these vessels will be equal to that of a vessel PBQFAK, the sides of which are plane surfaces, parallel to the masts.

The propositions immediately preceding, relate to the conic or Apollonian parabola: they have been inserted, with a view of establishing and extending the theory of stability. It may also be remarked, that the sides of vessels are in some instances constructed nearly of these forms; for the same reasons, it may be not altogether useless, to examine on what principle the stability of vessels is to be investigated, when the forms of the sections are parabolic curves of the higher orders, such as are represented in fig. 23. The line *c*BCO is a conic or Apollonian parabola, *d*BDO is a cubic, and *e*BEO a biquadratic parabola. *f*BFO (fig. 23.) is a parabola of 8 dimensions, and *g*BGO a parabola of 50 dimensions, which are drawn from a geometrical scale, in order to give a true representation of the forms of these curves.

The general equation, determining the relation between the abscissæ and ordinates of any parabola, of the dimensions n, is $y^n = p^{n-1} \times x$, if the ordinates are drawn perpendicular to the axis of the curve; and $y = a + px + qx^2 + rx^3$, &c. $+ vx^n$,

if the ordinates are drawn parallel to the axis; y and x signifying the ordinate and corresponding abscissa; the other letters denoting constant or invariable quantities, to be determined by the properties of the figures. In these figures, it is observable, that the breadths toward the vertex O, are always greater in the curves which are of the higher dimensions; and, as the dimensions are continually increased, the figure approaches more nearly to a rectangular parallelogram,* with which it

* The radius of curvature of the conic parabola at the vertex (fig. 23.) is half the principal parameter; but, in all the parabolas of the higher orders, the radius of curvature at the vertex is infinite. Suppose x to represent the abscissa, or distance of the ordinate y from the vertex, measured along the axis of the curve: as x increases from 0, the radius of curvature decreases till it becomes a minimum, and then increases: a difficulty seems to arise respecting the magnitude and variation of the radius of curvature, when, the dimensions being increased *sine limite*, the form of the curve approaches continually, and ultimately coincides with, the rectangular parallelogram. If the equation of the curve be $y^n = p^{n-1} \times x$, where p represents the parameter, the radius of curvature of the curve at the extremity of an ordinate y, of which the abscissa is x, will be found $= p \times \dfrac{\left(n^2 \times x^{\frac{2n-2}{n}} + p^{\frac{2n-2}{n}}\right)^{\frac{3}{2}}}{n \times \overline{n-1} \times p^{\frac{n-1}{n}} \times x^{\frac{n-2}{n}}}$, which quantity is a minimum when $x = p \times \left(\dfrac{n-2}{2n^3 - n^2}\right)^{\frac{n}{2n-2}}$: consequently, the least radius of curvature itself, or $r = p \times \dfrac{\left(\dfrac{3n-3}{n-2}\right)^{\frac{3}{2}}}{\dfrac{n-1}{n^2} \times \left(\dfrac{2n^3-n^2}{n-2}\right)^{\frac{2n-1}{2n-2}}}$; and, when n is increased *sine limite*, the abscissa corresponding to the least radius of curvature, or $x = p \times \dfrac{1}{\sqrt{2} \times n}$, and the least radius itself, or $r = p \times \dfrac{\sqrt{27}}{2n}$, both of which quantities are evanescent, shewing that if the dimensions of the parabola are increased *sine limite*, the curvature at the extremity of the ordinate, when the abscissa $= 0$, is infinite, the radius of curvature being nothing, as it ought to be, at the point H of the parallelogram BHOD,

ultimately coincides, when the dimensions are increased *sine limite*. This extreme case has relation to the subject of stability: for, whatever may be the effect of giving to the sides of ships the forms of the several higher orders of parabolas, it is certain, that as the dimensions of these curves are increased, the stability will approach to that which is the consequence of making the sides parallel to the masts; but it has been shewn, that when the sides coincide with the form of a conical para-

considered as a parabolic curve of infinite dimensions, the two portions of the curve BH, HO, (fig. 23.) being inclined at a right angle, when coincident with the sides of a rectangular parallelogram: but, since the curvature is nothing at the vertex O, the abscissa being then $= 0$, and before the abscissa has increased to any finite line, the curvature at the extremity of the corresponding ordinate OH is infinite; and since the curvature between the points O and H must necessarily pass through all the intermediate gradations of magnitude, it becomes a question to define the abscissa and corresponding ordinate, when the radius of curvature is a finite line: 2dly, when it becomes evanescent; and, lastly, when it is again infinitely great. By referring to the preceding expressions for the abscissa and corresponding radius of curvature, it is found, that if p represents the parameter, and x is made $= \frac{p}{n^3}$, (the number n denoting the dimensions of the curve,) when n is increased *sine limite*, the radius of curvature will be greater than any line that can be assigned: and such is the curvature of any portion of the line OH, between the points O and H. 2dly, if x is $= \frac{p}{n^2}$, the radius of curvature will be $= p$, the ordinate approximating to equality with the line OH. 3dly, if $x = \frac{p}{n}$, the radius of curvature will be smaller than any finite line: and, lastly, if $x = p$, or any finite line, the radius of curvature will be greater than any assignable line: which conclusions are immediately inferred from the equation expressing the radius of curvature, or $r = p \times \frac{\left(n^2 \times x^{\frac{2n-2}{n}} + p^{\frac{2n-2}{n}}\right)^{\frac{3}{2}}}{n \times \overline{n-1} \times p^{\frac{n-1}{n}} \times x^{\frac{n-2}{n}}}$, when the number of dimensions n is increased *sine limite*, these successive changes in the radius of curvature taking place while the abscissa x is increased from 0 to any finite magnitude.

bola*, the stability† is the same as when the sides are plane surfaces, parallel to the plane of the masts. It is inferred that if the sides of a vessel are formed to coincide with a parabola of the lowest, and the sides of another vessel with a parabolic curve of the highest dimension, all the other conditions being the same, the stabilities of the two vessels will be equal in these two extreme cases.

In proceeding to ascertain the stability of vessels, the vertical sections of which coincide with any parabolic curve, the rigid strictness of geometrical inference cannot be well preserved, when the oblique segments are objects of consideration, on account of the complicated properties of the figures.‡ But, in these and similar cases, methods of approximation may be employed, by which the stability corresponding to any given figure of the sides may be inferred, to a degree of exactness exceeding any that can be necessary in practice. These methods of approximation are either such as are required for the mensuration of curvilinear areas, or geometrical constructions which exhibit the lineal measures of stability not strictly and rigidly true, but approaching, as nearly as may be desired, to the true and correct measures.

The methods of approximation to be used for the quadrature of curvilinear spaces, are founded on Sir ISAAC NEWTON'S discovery of a theorem, by which, from having given any

* Case XI. pages 255, 256.

† The comparative stability, in this and similar observations, is understood to imply, that the vessels are inclined at equal angles of inclination from the upright, all the other conditions (the shape of the sides excepted) being the same.

‡ The areas of any parabolic segments, either direct or oblique, are geometrically quadrable, but, in the oblique segments, the positions of the centres of gravity are not determinable generally by direct methods.

number of points situated in the same plane, he could ascertain the equation to the curve which would pass through them all: and, by means of this equation, was enabled to express the ordinate in the curve, corresponding to an abscissa of any given length, as well as the area intercepted between any two of the ordinates. This discovery the author himself considered amongst his happiest inventions. Amongst the various uses of this theorem, that of determining by approximation the areas of curvilinear spaces is not the least considerable: for, by this means, the fluents of fluxional quantities, not discoverable by any known rules of direct investigation, are found, to a degree of exactness fully sufficient for any practical purpose, and with very little trouble of computation.

Mr. STIRLING, in his treatise intitled *Methodus differentialis*, has inserted a table for measuring curvilinear spaces terminated by parabolic curves, from having given 3, 5, 7, or 9 equidistant ordinates, and the abscissæ on which they are erected. The measures of the areas thus obtained are, under certain conditions hereafter stated, not approximations, but geometrically and strictly correct: the approximate values of curvilinear spaces, in general, are obtained from finding the correct areas terminated by parabolic lines which nearly coincide with the said curves, by passing through the extremities of the same ordinates.

The subjoined table contains Mr. STIRLING'S rules for expressing the areas of curvilinear spaces, from the conditions which have been mentioned; also additional rules for measuring the areas which are included between the extremes of 2, 4, 6, or 8 equidistant ordinates: the whole of this table has been recomputed and verified.

TABLE OF AREAS.

Number of equidistant ordinates.	Areas.
2	$\frac{A}{2} \times R$
3	$\frac{A + 4B}{6} \times R$
4	$\frac{A + 3B}{8} \times R$
5	$\frac{7A + 32B + 12C}{90} \times R$
6	$\frac{19A + 75B + 50C}{288} \times R$
7	$\frac{41A + 216B + 27C + 272D}{840} \times R$
8	$\frac{36799A + 175273B + 64827C + 146461D}{846720} \times R$
9	$\frac{989A + 5888B - 928C + 10496D - 4540E}{28350} \times R$

In this table, the letter A denotes the sum of the first and last ordinate of the number opposite to it in the first column: B is the sum of the second and last but one: C is the sum of the third and last but two, and so on. The extreme letter, suppose D, (as in the rule opposite 8 ordinates,) is the sum of the two middle ordinates, if the number of ordinates is even; or the extreme letter, suppose D, (as in the rule opposite 7 ordinates,) is the middle ordinate alone, if the number of ordinates is odd. R is the entire length of the abscissa, which is always equal to the common interval between the ordinates, multiplied by the number of ordinates diminished by unity.

Let the area to be measured be terminated by the curve line ABCD, *&c.* (Tab. XIII. fig. 24.): A′I′ is an abscissa, on which a number of equidistant ordinates AA′, BB′, CC′, *&c.* are erected at right angles. If ABCD, *&c.* represents a parabolic line of any dimension, suppose n, the relation between the ordinates and ab-

scissæ being expressed by the equation $y = a + px + qx^2 + tx^3$, &c. $+ ux^n$, (in which case, the ordinates are drawn parallel to the axis of the curve,) a measure of the area contained between the extremes of $n + 1$ ordinates will be obtained with geometrical exactness, by computing from the rule in the table which is opposite the number of ordinates $n + 1$, supposing the table to extend to that number: but if, as it usually happens in cases which practically occur, that the nature of the curve is unknown, or the conditions in other respects different from those which are required for the mensuration of the area with perfect correctness, it becomes a question, which particular rule in the table should be adopted for inferring an approximate value of the area, since an exact quadrature is not obtainable. For this purpose, there are several reasons for preferring the rules opposite the number of ordinates 2, 3, and 4 to the others, which require a greater number of ordinates; the common distance between them being the same. In the first place, the rules here pointed out are far less troublesome in the application; a circumstance which ought to have weight, although of less importance than another consideration, which is, that the results derived from these rules, particularly from the two latter, will in general approximate as nearly to the true value, sometimes more nearly, than those which are obtained by calculating from the other more complicated theorems, unless the curve should happen to be such as admits of being correctly measured by any of the rules requiring a greater number of ordinates; a circumstance not likely to occur in practical mensurations.

Let it be proposed to measure by approximation any curvilinear space AA′I′IA (fig. 24.) For brevity, let the successive ordinates AA′, BB′, CC′, *&c.* be denoted by the letters a, b, c,

&c. respectively; also, let the common distance between the ordinates (fig. 23.) or $A'B' = B'C' = C'D'$ be $= r$: according to the theorem for measuring the area contained between two ordinates, or $\frac{A}{2} \times R$, the curve line AB is supposed to coincide with the right line AB which joins the extremities of it; the space measured by this rule is the trapezium AA′B′BA; and, since $A = a + b$, and $R = r$, the area of the trapezium, or $\frac{A}{2} \times R = \overline{a + b} \times \frac{r}{2}$.

According to the rule opposite 3 ordinates, the curve line ABC is supposed to coincide with a portion of the conic parabola, the axis of which is parallel to the ordinates. And since, by this rule, the area $= \overline{\underline{A + 4B}} \times \frac{R}{6}$, in which expression $A = a + c$, $B = b$, and $R = 2r$; by substituting these values, the area $AA'C'CA = \overline{a + 4b + c} \times \frac{r}{3}$. If the curve ABC should actually be a portion of the conic parabola, the given ordinates being parallel to the axis of the curve, the area AA′C′CA will be measured by this rule with exactness absolutely perfect; and, the more nearly the curve which terminates the area approaches to the form of the conic parabola, the more nearly will the result of calculating by this rule approximate to the true value of the area. But it is evident, that since in this approximation, the arc of a conic parabola is drawn through the points A, B, C, being the same points which terminate the ordinates of the given curve, the difference between the parabolic area and that which is given must, in most (except extreme) cases, be next to an insensible quantity, when applied to practical mensuration.

In the mensuration of areas by the rule opposite 4 ordinates,

a parabolic curve line is supposed to be drawn through the points A, B, C, D, of the 3d dimension, such as the arc of a cubic parabola, the ordinates of which are parallel to the axis of the curve, and the area terminated by this curve line is assumed to approximate to the given area AA′D′DA: by this rule, the area $= \overline{A + 3B} \times \frac{R}{8}$, in which expression $A = a + d$, $B = b + c$, and $R = 3r$, which being substituted for their respective values, the area AA′D′DA $= \overline{a + 3b + 3c + d} \times \frac{3r}{8}$.

In order to bring these rules into a form convenient for practical use, let it be proposed to measure the area AA′G′GA (fig. 24.) intercepted between the extremes of 7 ordinates.

1st. Suppose the right lines AB, BC, CD, *&c.* to be assumed, instead of the curve lines AB, BC, CD, *&c.* as terminations of the space to be measured: then the area AA′G′GA will be equal to the sum of six trapeziums; AA′B′B, BB′C′C, CC′D′D and so on.

The area of the trapezium AA′B′B $= \overline{a + b} \times \frac{r}{2}$, by the rule opposite 2 ordinates: by the same rule, the area of the trapezium BB′C′C $= \overline{b + c} \times \frac{r}{2}$; the area of the trapezium CC′D′D $= \overline{c + d} \times \frac{r}{2}$, and so on. By adding these six separate areas, the sum will be the area of the space AA′G′GA $= \overline{a + 2b + 2c + 2d + 2e + 2f + g} \times \frac{r}{2}$. The law of continuation for a greater number of ordinates is obvious. This rule is precisely the same with that which is given by M. BOUGUER, in his work entitled "*Traité du Navire,*"* under a form somewhat different: his rule is this; from the sum of all the ordinates subtract $\frac{1}{2}$ of the sum of the first and last; the

* Page 112.

result multiplied into the common distance between the ordinates will be the exact area of the figure, considered as consisting of trapezia, and an approximate value of the curvilinear area in which the said trapezia are inscribed. This rule he professes not to be a very correct approximation, but such as may be deemed sufficient for most practical mensurations. It must be acknowledged, that in mensurations independent of others, the errors arising from this rule are often not considerable, (in many cases they are very small;) but, considering that in naval mensurations, areas obtained by approximation are necessarily the data from which other results are to be inferred, also by approximation, a doubt may arise whether the errors thus accumulated may not, in some cases, become too great; at least it may not be improper to be provided with rules which may be relied on, as approximating more nearly to the true measures of areas.

Let the same area be measured by the rule opposite 3 ordinates, according to which it is supposed that the curve line ABC coincides with the arc of the conic parabola. By this rule, the area $AA'C'CA = \overline{a + 4b + c} \times \frac{r}{3}$: also the area $CC'E'EC = \overline{c + 4d + e} \times \frac{r}{3}$; and the area $EE'G'GE = \overline{e + 4f + g} \times \frac{r}{3}$: adding these three areas together, the sum is the area $AA'G'GA = \overline{a + 4b + 2c + 4d + 2e + 4f + g} \times \frac{r}{3}$.

This rule is the same with that which Mr. SIMPSON has demonstrated in his Essays, page 109, from the properties of the conic parabola, perhaps not noticing that it was to be found in Mr. STIRLING's table of areas.

Mr. CHAPMAN, an eminent author on the subject of naval architecture,* applies this theorem to naval mensurations, as a substitute, and certainly an useful one, to the less perfect rules which are employed for this purpose, in the works of M. BOUGUER and other authors. The example by which Mr. CHAPMAN illustrates the use of this rule is the same with that which is given in Mr. SIMPSON'S Essays.

This approximation to the measures of areas being applicable

* The following observation on this theorem is inserted in the Report from the Committee of the French Royal Marine Academy, who were appointed to examine the translation of Mr. CHAPMAN'S Treatise on Naval Architecture, by M. VIAL DE CLAIRBOIS; this report is prefixed to the French edition of Mr. CHAPMAN'S work.

"Ce célèbre constructeur commence par donner une nouvelle méthode de calcul de "déplacement, qui sans être beaucoup plus longue que celle que l'on emploie com- "munément, donne un résultat infiniment plus exact. On considére ordinairement "les parties curvilignes des plans de flottaisons, ou de gabarits, entre les extrémités "des ordonnées, comme des droites; M. CHAPMAN les regarde comme des parties "paraboliques; et, de la nature de cette section conique, et du trapeze, il tire une "expression sur laquelle il fonde un calcul assez simple," *&c.*

A comparison of the results derived from this rule, and from that which is employed by M. BOUGUER, does not seem to confirm the opinion of the very superior exactness which the committee here attribute to the former rule: that it is more exact there is no doubt, especially when the curvature is at all irregular in respect to its variation, and the results inferred are data on which other computations are to be founded; but, in many of the cases which occur in practical mensurations, the latter rule approximates to the required results sufficiently near the truth, as will appear by the instances in the subsequent pages. The expression "une nouvelle méthode" cannot be understood to mean a rule of computation newly invented, but one which Mr. CHAPMAN has first applied to naval mensurations. In this sense, the theorem inserted in the 265th and 268th pages of these papers would be entitled to the appellation of "a new method:" but it has already been shewn, that the three rules here described, and employed in the computations which follow, are only particular cases of the general method demonstrated in the works of Sir I. NEWTON, STIRLING, SIMPSON, and other authors.

only when the number of given equidistant ordinates is odd, to obtain the area when the number of ordinates is even, another rule, to be employed either singly, or in conjunction with the former, may be selected from Mr. STIRLING's table of areas. It is that which stands opposite 4 ordinates.

Let it be proposed to measure the area AA′G′GA. According to this rule, the area AA′D′DA $= \overline{a + 3b + 3c + d} \times \frac{3r}{8}$

also the area DD′G′GA $= \overline{d + 3e + 3f + g} \times \frac{3r}{8}$

these two areas being added together, the sum will be the area AA′G′GA $= \overline{a + 3b + 3c + 2d + 3e + 3f + g} \times \frac{3r}{8}$.

When the number of given equidistant ordinates is small, these theorems will be most conveniently used in the forms here given; but, when the ordinates are numerous, the trouble of arithmetical computation will be considerably abridged, by employing them according to the general rules inserted underneath.

These theorems for approximating to the values of areas, may be applied, with advantage, to the integration of fluxional quantities, the fluents of which cannot be obtained by direct methods; or, if obtained, requiring very long and troublesome calculations.*

Suppose z to represent the abscissa of a curve, on which the or-

* On this principle, the rules of approximation here given are applicable to determine the positions of the centres of gravity, both of areas and solid spaces. If y is put to represent the ordinate erected perpendicular to an abscissa, at the distance z from the initial point thereof, the fluent of $yz\dot{z}$ (fig. 24.) will be the sum of the products arising from multiplying each ordinate into the small increment $\dot{z}$, and also into the distance z from the initial point. And, since the area intercepted between the ordinates AA′ and y is the fluent of $y\dot{z}$, it follows, that the distance of the centre of gravity of this curvilinear space from the ordinate AA′, measured on the abscissa A′I′, is $= \frac{\text{fluent } yz\dot{z}}{\text{fluent } y\dot{z}}$. The approximate values of these fluents are obtained from the Rules I. II.

dinates (expressed by Z, a general term or function of z), a, b, c, d, *&c.* are erected at right angles, and at intervals each of which is $= r$, so that when $z = o$, $Z = a$: when $z = r$, $Z = b$: when $z = 2r$, $Z = c$, and so on. If innumerable ordinates or values of Z be supposed drawn between each of those which are given, at the common very small interval $\dot{z}$, the sum of the products arising from multiplying each of the ordinates into the increment $\dot{z}$, that is, the fluent of $Z\dot{z}$, will be found, by approximation, according to the three following rules; which may be not improperly termed, rules for approximating to the integral values of fluxional quantities. According to

RULE I.

$$\text{Fluent of } Z\dot{z} = \overline{P - \tfrac{S}{2}} \times r;$$

in which expression,

$P =$ the sum of all the ordinates $a + b + c + d$, &c.
$S =$ the sum of the first and last ordinate.
$r =$ the common distance between the ordinates.

RULE II.

$$\text{Fluent of } Z\dot{z} = \overline{S + 4P + 2Q} \times \tfrac{r}{3};$$

in which expression,

$S =$ the sum of the first and last ordinate.
$P =$ the sum of the 2d, 4th, 6th, 8th, *&c.* ordinate.
$Q =$ the sum of the 3d, 5th, 7th, 9th, *&c.* ordinate, (the last excepted.)
$r =$ the common distance between the ordinates.

and III.; and the positions of the centres of gravity are thus determined according to the methods of computation employed in the subsequent pages. The position of the centre of gravity in solid bodies, is determined by a similar application of these rules.

RULE III.

$$\text{Fluent of } Z\dot{z} = \overline{S + 2P + 3Q} \times \frac{3r}{8};$$

in which expression,

S = the sum of the first and last ordinate.

P = the sum of the 4th, 7th, 10th, 13th, &c. ordinate, (the last excepted.)

Q = the sum of the 2d, 3d, 5th, 6th, 8th, 9th, &c. ordinate.

r = the common distance between the ordinates.

It is to be observed, that the first of these rules approximates to the fluent, whatever be the number of given ordinates. The second rule only requires that the number of ordinates shall be odd. To apply the third rule, it is necessary that the number of ordinates given shall be some number in the progression 4, 7, 10, 13, &c. that is, the number of ordinates must be a multiple of 3 increased by unity. But, in every case, the approximate fluent may be obtained, either from the Rule II. or the Rule III. or by employing both rules conjointly.

Before these theorems are applied to practical mensurations in naval architecture, it may be satisfactory to examine, by a few trials, to what degree of exactness they approximate to the correct values of curvilinear spaces. This will be known, if the area of some curve, which is exactly quadrable by other geometrical rules, be measured by them. Such as a parabolic figure of which the equation is $y^8 = p^7 x$, x being the abscissa coincident with the axis, and y the corresponding ordinate perpendicular to it.

The semi-area of this parabola (fig. 25, 26.) is known to be $xy \times \frac{8}{9}$; and the curve is termed a parabola of 8 dimensions.

OD is an abscissa, being a portion of the axis of this curve, and the parameter is assumed = OD. If, therefore, an ordinate BD, or DA, is drawn through the point D perpendicular to DO; the lines DB, DO, and DA, will all be equal. Let BD be divided into 10 equal parts,* considered in this instance as an abscissa, on which, at the points of division, the several ordinates are erected perpendicular to BD, denoted in the figure by the letters *a, b, c, d, &c.* If BA is assumed = 100 equal parts, DB, DO, and DA, are each = 50, and the common interval between the ordinates = 5; the numerical values of the successive ordinates *a, b, c, &c.* are expressed in the annexed table.

a	= 50.0000
b	= 50.0000
c	= 49.9999
d	= 49.9967
e	= 49.9672
f	= 49.8047
g	= 49.1602
h	= 47.1175
i	= 41.6113
k	= 28.4766
l	= 0.0000
Sum of all the ordinates	= 466.1341

According to the Rule 1. making the sum of all the ordinates, or P = 466.1341, the sum of the first and last ordinate, or S = 50 : r = 5 = the common distance of the ordinates.

The area BDO $= \overline{P - \frac{S}{2}} \times r$

$= 2205.670$

correct area $= \frac{50 \times 50 \times 8}{9} = 2222.222$

Difference † or error of the approximation $= 16.552$

* Any line being assumed in a curve as an abscissa, lines drawn parallel to each other, and intercepted between the abscissa and the curve, are termed ordinates. To exemplify these rules for approximating to the areas of curvilinear spaces, it was necessary to consider the ordinates as being drawn in some cases parallel, and in others perpendicular, to the axis.

† It must not be concluded, from this instance, that the errors in measuring curvilinear areas by the Rule 1. will be usually so great as 16 parts in 2222. In applying the Rule 1. to this parabolic space, the quick variation of curvature in some parts of the curve, causes the space so measured to deviate more from the truth than would happen in ordinary cases, such as commonly occur in practical subjects. If, instead of

If the same area is measured by the Rule II. the process will be as underneath:

Sum of all the ordinates	-		= 466.1341
Sum of the first and last, or	S =	50.0000	
Sum of the 2d, 4th, 6th, &c. or	P =	225.3955	
	S + P =	275.3955	275.3955
Sum of the 3d, 5th, 7th, &c. (except the last) or	Q =		190.7386

and r being = 5, the area BDO = $\overline{S+4P+2Q} \times \frac{r}{3}$ = 2221.765

Correct area	-	= 2222.222
Difference or error of the approximation	=	·457

Let the same area be measured by the Rule III. the area DOKI, between the two ordinates a and b, being $= 5 \times 50 = 250$, the remaining area, from the ordinate b to the ordinate $l = 0$, will be obtained from the following computation:

The sum of all the ordinates		= 416.1341
Sum of the first and last, reckoning b the first, and the last $l = 0$ or S	= 50.0000	
Sum of the 4th, 7th, 10th, &c. (b being the 1st.) or P - -	= 97.0847	147.0847
Sum of the 2d, 3d, 5th, 6th, &c. from b, or Q		= 269.0494

the total area = DBO, (fig. 25.) a portion of it, which is contained between the ordinates a and g, should be measured, the area, computed from either of the three rules, would deviate very little from the truth, as appears from the following results.

Areas contained between the ordinates a and g,

computed by	Rule I.	Rule II.	Rule III.
Areas	1496.74	1497.17	1497.13
Correct area	1497.20	1497.20	1497.20
Difference or error of approximation -	.46	.03	.07

If the rule in the table of areas opposite 7 ordinates should be applied to measure the area between the ordinates a and g, (fig. 25.) the result would be geometrically correct.

And, since $r = 5$, the area between the ordinates

b and $l = \overline{S + 2P + 3Q} \times \frac{3r}{8}$ -	$= 1971.220$
Area IKOD between the ordinates a and b	$= 250.000$
area BDO - -	$= 2221.220$
Correct area BDO - -	$= 2222.222$
Difference or error of the approximation	$= 1.002$

In applying these rules, it is necessary to observe, that if the ordinates are drawn perpendicular to the axis of the curve, whenever the area to be measured, or any part of it, is adjacent to the vertex O, the area found by these rules will be the least exact: in such cases, it will be requisite to assume an abscissa near the vertex O, perpendicular to the axis: by erecting equidistant ordinates upon it, parallel to the axis, the area will be found, with the same exactness as in the other cases, which will appear by the following computations.

DOA (fig. 26.) is a semiparabola, similar and equal to DOB.

Let the line DO = 50 be divided into 10 equal parts, each = 5; and, through the points of division, let the successive ordinates b, c, d, &c. be drawn perpendicular to DO: according to the preceding observations, if the entire area DOA should be computed by either of the three rules, the result would be less exact than in the former cases. To obtain an approximate value of the area, sufficiently near the truth, a portion of the area adjacent to the vertex O, suppose XEO, is to be separately computed. If OX be = 10, the line XE will = 40.8890, which being divided into six equal parts, each of them will = 6.815: let the ordinates p, q, s, t, *&c.* be erected at the points of division, perpendicular to XE:

p	=	10.0000
q	=	10.0000
s	=	9.9985
t	=	9.9805
u	=	7.6750
v	=	9.6100
w	=	0.0000
	=	57.2640

From these ordinates, the area OXE is found by the Rule II. to be $159.8390 \times \frac{6.815}{3} = 363.101$.

For obtaining the area DAXE, the ordinates *a, b, c, d, &c.* erected on the abscissa DX, are as expressed underneath:

a	=	50.000
b	=	49.346
c	=	48.624
d	=	47.820
e	=	46.907
f	=	45.851
g	=	44.590
h	=	43.014
i	=	40.889
Sum of all the ordinates	=	417.041
k	=	37.495
l	=	0.000

The area DAXE, between the ordinates *a* and *i*, is found by the Rule II. to be - - = 1858.760

The area between the ordinate *i* and *l*, before found = 363.101

*Entire area DOA = 2221.861

Correct area - = 2222.222

Difference or error of the approximation - = .361

* If the area between the ordinate *a* and *i* had been computed by the Rule I. the result would have been nearly the same.

Area between the ordinates *a* and *i*, by Rule I, is -	1857.98
Area between the ordinates *i* and *l*, before found, is	363.10
Entire area DOA - -	= 2221.08
Correct area - - -	= 2222.22
Difference or error of the approximation - -	1.14

If the total area DOA (fig. 26.) should be measured by one computation, suppose from Rule II. the

Area would be found - - = 2176.875

Correct area - - - - = 2222.222

Difference or error of the approximation - = 45·347

If this area should be computed according to the Rule I. the error of the approximation will be found - - - - - = 74·54

It is easily shewn, that these errors, which are far from inconsiderable, arise almost wholly from the mensuration of the area OXE, (fig. 26.) adjacent to the vertex O. For, by measuring that area, according to the Rule II. from three ordinates $l = 0$, $k = 37·495$, $i = 40.889$, the area is found to be 318.115

Whereas the correct area OXE $= 10 \times 40.889 \times \frac{8}{9} =$ 363·457

Difference or error from computing the area OXE by Rule II. - - - - = 45·342

Scarcely differing from - - = 45·347

which was found to be the error from computing the entire area DOA by this rule.

If the area between the ordinates a and k be measured by the Rule III. it is found to be - - = 2055·390

Area between the ordinate k and l, by the properties of the figure, is $= 37·495 \times 5 \times \frac{8}{9}$ - = 166.640

Entire area DOA - = 2222.030

Correct area - - = 2222.222

Difference or error of this approximation = .192

From these computations it is evident, that the rules here given, when employed with attention to the necessary limita-

tions and restrictions, will approximate to the measures of areas, to a degree of exactness fully sufficient for naval mensurations; and further, will be useful in determining by approximation the integral values of fluxional quantities in general, especially those which occur in the investigation of practical subjects.

CASE XII.

Still supposing the vertical sections of a vessel to be equal and similar figures, let BOA (fig. 27.) represent one of these sections; the figure being either a curve of the higher dimensions, or a curve not formed according to any geometrical law, of which the lengths of the ordinates, and of any other lines given in position, are supposed to be measurable, and given in quantity: the angle at which the vessel is inclined from the upright, and the other necessary conditions being known, it is required to find, by geometrical construction, a line which shall approximate nearly to the measure of the vessel's stability.

1st Method.

BA represents the intersection of the water's surface when the vessel floats upright: bisect BA in the point D; and, through D, draw the line NDM inclined to the line BA at an angle ADM, equal to the given angle of the vessel's inclination: let the area of the figure ADMb, also the area of the figure BDNc, be found by means of the rules which have been described: suppose the area ADMb to be greater than the area BDNc, and let E represent the difference between them: from

the point D, in the line DA, set off a line * DS $= \frac{E}{NM \times \sin. ADM}$: a line CSH, drawn through the point S, parallel to NM, will cut off the area ASH*b*A, very nearly equal to the area BSC*c*B; (fig. 27.) consequently, when the vessel is inclined to the given angle, the water's surface will intersect the vessel in the line CSH. (Tab. XIV. fig. 28.) Draw the lines AH, BC. Let M and I be the centres of gravity of the triangles ASH, BSC, respectively; through M and I, draw M*l*, I*k*, perpendicular to CH; through *b*, † the centre of gravity of the area A*b*H, draw *b*U perpendicular to SH; and, through *c*, the centre of gravity of the area B*c*C, draw *c*R perpendicular to CH: in the line *l*U, take *l*L to LU as the area AH*b* is to the area ASH*b*; and, in the line *k*R, take *k*K to *k*R as the area B*c*C is to the area BSC*c*. Let G be the centre of gravity of the vessel; and let E be the centre of gravity of the displaced volume when the vessel floats

* Let the area ASH*b* be supposed equal to the area BSC*c*, (fig. 27.) and make either of them = A. Let the space DMHS be denoted by M, and the space NDSC by N: then the area ADM*b* will approximate very nearly to the quantity A + M, and the area BDN to A − N. The difference of these areas will be M + N, which is equal to the area NMHC = E = MN × DY; and, consequently, DY $= \frac{E}{MN}$; and, because DY : DS : : sin. DSY, or ADM to radius, it will follow that DS $= \frac{E}{MN \times \sin. ADM}$.

† In these small curvilinear spaces, it will be sufficient to assume the positions of the centres of gravity by estimation, on a supposition that the curve coincides with the arc of a common parabola; in which case, the centre of gravity is situated at the distance of $\frac{2}{5}$ of the abscissa from the ordinate, or chord which joins the extremities of the curve. The position of the abscissa is determined by drawing chords parallel to the given chord, and by drawing a line through the points which bisect the several chords. But, when the curvilinear spaces AH*b* are extremely small, as represented in this figure, (fig. 28.) no sensible difference in the result will ensue, whether the line *b*U is drawn through the centre of gravity of this curvilinear space, or through any other point which is adjacent to that centre.

upright. Through the point E, draw EV parallel and equal to KL: and, in EV, take ET to EV as the area ASH*b* is to the area representing the entire volume displaced: through G, draw GU parallel to CH; and, through T, draw TZ perpendicular to GU, intersecting the line GU in the point Z. GZ is the measure of the vessel's stability.

2d Method.

Let BOA (fig. 29.) be the given vertical section of a vessel, intersected by the water's surface BA when floating upright. G is the vessel's centre of gravity: E is the centre of gravity of the volume displaced in the upright position. Let the area BOA be measured by either of the three Rules, suppose Rule 1.; and through D, the bisecting points of BA, draw NDM inclined to the line BA in the angle ADM, equal to the given inclination of the vessel from the upright. Let the area NOAM be measured, by erecting equidistant ordinates on the line MN. If the area, so found, is equal to the area BOA, the area DBN will be equal to the area ADM. But, if they are unequal, let the difference be represented by E, and from D, toward the largest of the areas, suppose ADM, set off DS $= \frac{\mathrm{E}}{\mathrm{NM} \times \sin.\ \mathrm{ADM}}$; and, through S, draw CSH parallel to NM. The area ASH*b* will approximate to equality with the area BSC*c*; and, consequently, when the vessel is inclined through the given angle ASH, it will be intersected by the water's surface in the line CH. On the line HC, let the equidistant ordinates *a*, *b*, *c*, *d*, *&c.* be erected perpendicular to CH; and let the common interval between the ordinates be $= r$. Let the measure of the area CLFK be obtained, and let *m* be the centre of gravity of this area: through the point *m*, draw *m*P perpendicular to CH: let each of the successive

given ordinates be multiplied into its perpendicular distance from the ordinate a. The terms resulting will be $a \times 0$, $b \times r$, $c \times 2r$, $d \times 3r$, *&c.*; let these terms be added together, and half the sum of the first and last term being subtracted from the amount, let the result be denoted by the letter C; Cr — area CLFK $\times$ KP will be the sum of the products arising from multiplying each evanescent area QX into its distance QK from the first ordinate a. In the line KH, set off a line KI* $= \frac{Cr - \text{area CLFK} \times \text{KP}}{\text{area COAH}}$. Through the point I, draw IT perpendicular to CH; and, through the vessel's centre of gravity G, draw GZ perpendicular to IT. GZ is the measure of the vessel's stability, when it is inclined from the upright through the angle ASH.

In the cases which have preceded, the vertical sections of vessels, or segments of vessels, are assumed as equal and similar figures: whereas, in reality, the form and magnitude of the sections are gradually changed, according as they intersect the longer axis at a greater or less distance from the head or stern. The solutions of the preceding cases, and the principles therein established, may be next applied to investigate the stability of vessels, taking into consideration the form and magnitude of each particular section intersecting the longer axis at right angles, and at equal distances; taking into account also, by the methods which have been described, all the sections intermediate between those which are given, that may be conceived to intersect the axis at a very small common interval.

* Since the Rule I. is employed in obtaining the value of the quantity Cr, according to this computation, the area COAH ought to be measured by the same rule; in which case, the line KI will be determined nearly with the same exactness as by either of the Rules II. or III.

CASE XIII.

The longer axis of a vessel is supposed to be divided into a given number of equal parts, and vertical sections to pass through the several points of division, intersecting the axis at right angles: the form and magnitude of each particular section being given, with the common distance between them, the positions of the centres of gravity of the vessel, and of the volume displaced, and the distance of the water-section from the keel, being known, it is required to construct the measure of the vessel's stability, when it is inclined from the upright through a given angle.

Let QBOAW (Tab. XV. fig. 30.) represent any vertical section of a vessel; suppose it to be the greatest or principal section: BA is the breadth of this section at the water-line, when the vessel floats upright: let CH represent the line which coincides with the water's surface, when the vessel is inclined from the upright through the given angle ASH. From the nature of the conditions, it is sufficiently evident that the point S, in any individual section, will not be determined on the same principle by which the position of that point was fixed according to the former solutions; that is, by making the area ASH equal to the area BSC; because, the volume immersed, and that which is caused to emerge in consequence of the vessel's inclination, will not now be proportional to these areas, as they* are on a supposition that the vertical sections are similar and equal figures. But, in the present case, the vertical sections being different, both in form and magnitude, the water's sur-

* See page 210.

face, intersecting the vessel in a plane passing through the line CH, when the vessel is inclined, will so divide the areas of the several sections, that although the area ASH*b* may not be equal to the area BSC*c*, in any of the vertical sections, yet the volume immersed, corresponding to, and included between, the areas of the figures ASH*b*, taken from the head to the stern of the ship shall be equal to the emerged volume which is included between the areas BSC*c*, in the several sections. Suppose the breadth of any section at the water-line* to be denoted by BA, and to be bisected in the point D. A vertical plane passing through the vessel's longer axis and the centre of gravity G,† and dividing the ship into two parts perfectly similar and equal, will pass through the points D, in all the sections: this plane may be termed the plane of the masts. It is easily shewn, that at whatever distance DS, from the middle point D, the plane of the water's surface, passing through the lines CH, intersects the line DA in any one section, when the vessel is inclined through the angle ASH, it will intersect the line DA at the same distance from the middle point D in all the other sections; that is, the distance DS will be the same in all the sections: for, by the supposition, the vessel is inclined round the longer axis, and consequently, the intersection of the two planes, passing through the lines BA and CH, will be parallel

* The same letters which are used to denote the several lines, in this vertical section, must be understood to represent the lines similarly drawn in each of the other sections. In the present instance, BA does not represent the breadth at the water's surface, of the principal, or any other individual vertical section, but represents generally that breadth, in any of the sections that may be referred to.

† Represented by the point G projected on the plane BOA. The centre of gravity E is, in like manner, here represented by projection on the same plane.

to the longer axis, and therefore parallel* to a line drawn through all the points D, from one extremity of the vessel to the other; the several lines DS are the perpendicular distances of these parallel lines, and are consequently all equal. In the next place, it is requisite to determine the magnitude of the line DS, according to the given conditions: whatever be the position of the points S, if lines CH are drawn through S, in each of the sections, inclined at an angle to the line BA, equal to the given angle of the vessel's inclination, the same plane will pass through all the lines CH. It is required to ascertain at what distance DS, from the points D, the plane CH, coinciding with the water's surface when the vessel is inclined, must pass, so as to cut off a volume on the side ASH*b*, being the volume immersed, which shall be equal to the volume in the side BSC*c*, which has emerged from the water, in consequence of the vessel's inclination.

In each section, through the middle point D, draw a line NDW, inclined to BA at an angle ADW, equal to the given angle of the vessel's inclination; the same plane will pass through the lines NDW, in all the sections. By the methods which have been described, let the area of the figure ADW*b* be measured in each section; from these equidistant areas, the solid contents of the volume between the two planes DA† and DW, and the side of the vessel intercepted, may be inferred by

* The points D being coincident with the water's surface, a line passing through them must be horizontal; and being, by the supposition, situated in the same plane with the longer axis, must therefore be parallel to it.

† Since the same plane passes through the lines DA, drawn coincident with the water's surface in all the sections, this plane may be supposed projected into the line DA on the plane DOA. For similar reasons, the line DW represents the plane which passes through all the lines DW in all the sections.

computing according to the Rules II. and III.; suppose this volume to be denoted by the letter P, and the volume contained between the planes DB, DN, and the side of the vessel, found by similar operations, to be denoted by the letter Q. Let the area* of the section of the vessel passing through the lines NDW be measured, from having given the lines NW in all the sections from the head to the stern, and let this area be put $= R$. If the volumes P and Q should be unequal, P being the greatest, in the line DA, set off, in each section, a line DS $= \frac{P-Q}{R \times \sin. ADW}$. If a plane CSH be drawn passing through all the points S, and inclined to the plane BA at the given angle of the vessel's inclination, the solid contents of the volume between the planes SA, SH, and the intercepted side of the vessel, will approximate to equality with the volume contained between the planes SB, SC, and the intercepted side of the vessel. Since, therefore, the water's surface coincides with the plane BA when the vessel is upright, when it is inclined round the longer axis, through the given angle ASH, the water's surface will intersect the vessel in the direction of a plane passing through the lines CH, in all the sections.

Let the solid contents of the volume immersed, or emerged, by the inclination, be denoted by the letter A.

In the section QBOAW, let M be the centre of gravity of the triangle ASH, and let *b* be the centre of gravity of the curvilinear area AH*b*; also, let I be the centre of gravity of the triangle BSC, and let *c* be the centre of gravity of the curvilinear area BC*c*: through these points, draw the lines

* That is, the area of the section coinciding with the water's surface, when the vessel is inclined to the given angle.

M*l*, *b*U, I*k*, *c*R, perpendicular to CH; and, in the line *l*U, take a line *l*L, which is to *l*U as the curvilinear area AH*b* is to the area ASH*b*. Through the points S, in all the sections, let a line F*f* be drawn perpendicular to SH; the same plane will pass through all these lines. LS will be the distance of the centre of gravity of the area ASH*b* from the plane F*f*. The products arising from multiplying each area ASH*b* into the distance SL, of its centre of gravity, from the plane F*f*, are to be calculated in all the sections; from which products, by means of the Rules* I. II. and III. the sum of the products arising from multiplying each evanescent solid, of which the base is the area ASH*b*, and the thickness a small increment of the axis, into the distance SL of its centre of gravity from the plane F*f*, will be obtained. The sum of these products, divided by the solid contents of the volume immersed A, will be the distance of the centre of gravity of that volume from the vertical plane F*f*. Suppose this distance to be equal to the line SQ: let the d istance PS, of the centre of gravity of the volume emerged, or BSC*c*, from the plane F*f*, be found from similar computations; the line PQ will be the distance of the centres of gravity of the volumes ASH*b*, BSC*c*, estimated in the direction of the line CH, perpendicular to the plane F*f*.

The solid contents of the entire volume displaced by the ship, are to be obtained from the areas, either of the vertical or horizontal sections.

* Whenever the Rules I. II. and III. are referred to, it is meant that the computation is to be made from one or more of these rules, according to the number of ordinates given, or as other circumstances may direct.

† See Appendix.

The ordinates drawn in the several sections being set down in regular order, the area of any horizontal section is to be found from the corresponding series of ordinates, by means of the Rules II. and III. and, by the same rules, from the areas of the horizontal sections so determined, the solid contents of the total volume immersed are to be inferred; some allowance being made for the irregular parts of the volume adjacent to the head and stern, if attention to these additional volumes should be thought necessary. That part of the volume which is contained between the keel and the nearest horizontal area, is obtained by first finding the area of each vertical section between the keel and nearest ordinate: from these areas, by means of the Rules II. and III. the solid contents of the volume between the keel and nearest horizontal section will be measured, and is to be added to the volume before found, which is contained between the two extreme horizontal sections.

Let the solid contents of the displaced volume be denoted by V.

From the areas of the horizontal sections, and the common interval between them, the distance DE, of the centre of gravity of the volume immersed, from the water-section, is to be obtained by means of the Rules II. and III.; by finding the sum of the products arising from multiplying each evanescent solid, of which the base is any horizontal section, and the thickness a small increment of the vertical axis, into that small increment, also into its distance from the water-section: the sum of these products, being divided by the solid contents of the volume displaced, will be the distance DE, of the water-section, from the centre of gravity of that volume.

The position of the vessel's centre of gravity G, depends

partly on the construction and equipment of the vessel, and partly upon the distribution of the lading and ballast, which circumstances therefore determine the distance GE, or the distance between the centre of gravity of the vessel, and that of the displaced volume.

These several conditions having been determined, the construction of the vessel's stability will be as in the former cases. Through the point E, draw the line EV parallel and equal to the line PQ; and, in EV, take ET to EV as the volume immersed by the inclination is to the entire volume displaced; or as A to V. Through the centre of gravity G, draw GU parallel to CH, and through the point T draw TZ perpendicular to GU. GZ is the measure of the vessel's stability, when inclined from the upright through the angle ASH.

The weight of the vessel and lading is found from the following proportion:* as 1 cubic foot is to V, the volume displaced, so is $\frac{1}{35}$ part of a ton to the vessel's weight, which will therefore be $= \frac{V}{35}$ ton.

The arithmetical operations required for ascertaining the sta-

* According to Mr. COTES, (Hydrostatics, page 73,) the specific gravity of sea water is $= 1.03$, when that of fresh water is $= 1$. And, since the weight of a cubic foot of rain water is 1000 oz. or $62\frac{1}{2}$ pounds avoirdupois, it will follow, that the weight of a cubic foot of sea water is $62.5 \times 1.03 = 64.375$ pounds avoirdupois. Mr. CHAPMAN, in his Treatise on the Method of finding the proper Area of the Sails for Ships of the Line, infers the weight of an English cubic foot of sea water to be 63.69 pounds avoirdupois. If an average between these results be taken, the weight of a cubic foot of sea water will be very nearly 64 pounds avoirdupois; and the weight of 35 cubic feet of sea water will be almost exactly one ton. According to the tables published by M. BRISSON, the specific gravity of sea water is 1.0263, when that of rain water is 1. By computing from this specific gravity, the weight of a cubic foot of sea water will be 64.14 pounds avoirdupois. 64 pounds avoirdupois is assumed as the average weight, in the ensuing computations.

bility of vessels, by the methods here described, are far from difficult, although they necessarily extend to some length; in order to give an illustration of these rules, by applying them to a particular vessel, I obtained, by favour of Messrs. RANDALL and BRENT, eminent constructors, a draught expressing the form and dimensions of a large ship,* built for the service of the East India Company. According to this draught, the vessel is divided into 33 vertical segments, by 34 sections, intersecting the longer axis at right angles, and at a common distance of 5 feet.†

The lengths of the ordinates entered in the annexed table‡ sufficiently define the form and magnitude of each of the 34 vertical sections; it will not therefore be necessary to represent their figures by separate drawings, since the constructions and calculations founded on them, for inferring results in any one section, are similar to those which are required in the other sections.

The greatest or principal section, which, according to this draught, intersects the longer axis at about 60 feet from the 1st section adjacent to the head, is represented by the figure BAO (fig. 31.): BA is the breadth at the water-line = 43.16 feet. BA is bisected in the point D; and DO, drawn through D perpendicular to BA, is the distance of the keel from the

* The ship CUFFNELLS.

† Mr. BRENT, jun. obligingly took the trouble, at my request, of delineating each of these sections on a large scale, and likewise of drawing and measuring the equidistant ordinates necessary for calculating the areas thereof, together with such additional lines as are required for constructing the measure of the vessel's stability, according to the principles delivered in the preceding pages.

‡ See Appendix.

water-section = 22.75 feet. The line DR = 22 feet, is divided into 11 equal parts, and, through the points of division, 12 ordinates* are drawn, parallel to the line BA, at the common distance of 2 feet.

The vessel is supposed to be inclined round the longer axis, at an angle of 30°, and the line NDW is drawn through the point D, inclined to BA, at an angle ADW = 30°: proceeding according to the solution which has been given, by measuring the line DW = 22.6 feet, DA = 21.58 feet, the area of the triangle ADW $= \frac{21.58 \times 22.6}{4} = 121.92$ square feet. Also, by mensuration, the line WA = 11.55: this line being divided into six equal parts, of 1.925 each, if ordinates are drawn at the points of division, perpendicular to the line WA, they are found to be as here stated.

	Ordinates. Pts. of a Foot.	Numbers.	Products.
a	= 0.00	1	0.00
b	= 0.15	4	0.60
c	= 0.30	2	0.60
d	= 0.43	4	1.72
e	= 0.38	2	0.76
f	= 0.23	4	0.92
g	= 0.00	1	0.00
		Sum	4.60

$\frac{1}{3}$ common interval = .642; and the area AWb = 4.6 × .642 = 2.95

By computing according to the Rule II. from the 7 ordinates given, of which the two extremes are = 0, the area of the curve space AWb = 2.95, which being added to the area ADW = 121.92, the area of the entire figure ADWb = 124.87. By similar calculations, the area of the figure BDNc is found to be = 133.68.

The areas of the figures ADWb and BDNc, being measured in each of the 34 vertical sections, are found to be as follows.

* The numerical measures of these lines are inserted in the table of ordinates; (see Appendix;) the numbers are entered in the 12th vertical section. It is not necessary to express them in the figure.

Vertical Sections	Areas of the Figures ADW*b* Square Feet.	Areas of the Figures BDN*c* Square Feet.
1	42.86	23.61
2	81.53	58.92
3	100.80	86.80
4	114.16	105.27
5	121.56	115.70
6	121.75	120.90
7	123.47	125.36
8	125.20	129.82
9	124.87	131.04
10	124.54	132.27
11	124.69	132.97
12	124.87	133.68
13	124.87	133.68
14	124.87	133.68
15	124.82	133.42
16	124.78	133.17
17	124.20	132.85
18	124.62	132.53
19	123.91	131.05
20	123.21	129.57
21	121.06	127.48
22	118.91	125.40
23	117.50	122.66
24	116.10	119.93
25	114.01	116.88
26	111.91	113.83
27	108.96	109.81
28	106.01	105.80
29	101.82	98.92
30	97.24	91.71
31	92.41	79.95
32	86.31	66.06
33	81.60	48.20
34	68.35	17.92
	3767.77	3700.84

The vertical sections intersect the longer axis at a common interval of 5 feet. By computing, according to the Rule III. from the areas of the 34 figures ADW*b* here given, together with the common interval of 5 feet, a result will be obtained, which approximates very nearly to the solid contents of the volume between the planes DA, DW, and the intercepted side of the vessel, throughout the entire length of it.

Making, therefore, according to Rule III. the sum of the first and last area - - $= S =$ 111.21

And the sum of the 4th, 7th, 10th, 13th, &c. area (except the 34th,) $= P =$ 1167.58

$S + P =$ 1278.79

Sum of all the areas - $=$ 3767.77

Sum of the 2d, 3d, 5th, 6th, &c. areas - $= Q =$ 2488.98

Solid contents of the volume between the planes DA, DW, and the intercepted side of the vessel $= \overline{S + 2P + 3Q} \times \frac{15}{8} =$ 18587.5 cubic feet.

From the 34 areas of the figures BDN*c*, as entered in the table, by computing according to the Rule III. the solid contents are obtained, of the volume between the planes DB, DN, and

the intercepted side of the vessel: for, observing the notation already described, and applied to the areas BDN*c*.

$$S = 41.53$$
$$P = 1188.83$$
$$Q = 2470.48$$

The solid contents of the volume between the planes DB, DN, and the intercepted side of the vessel,

is $= \overline{S + 2P + 3Q} \times \frac{15}{8} = 18432$

Volume between the planes DA, DW, and the intercepted side of the vessel $= 18587$

Difference - $=$ 155 cubic feet.

The area of the section passing through all the lines CDW, is found from having these lines given by mensuration, and the common interval of 5 feet between them. This area is $= 7106$ square feet. The distance DY * between the plane NW and the plane CH, which coincides with the water-section, when the vessel is inclined 30° from the upright, is $= \frac{155}{7106} = .022$ parts of a foot, and the distance DS † $= \frac{.022}{\text{sin. } 30^\circ} = .044$ parts of a foot, or little more than $\frac{1}{2}$ an inch: a quantity of which it is unnecessary to take any account in this construction. ‡

When, therefore, the vessel is inclined from the perpendicular, through an angle of 30° round the longer axis, the water's surface will pass through the middle point D of the line BA, in all

* Page 283. † Ibid.

‡ The form of the sides above and beneath the water line, in this vessel, causes the points D and S almost to coincide; but, in vessels differently formed the distance of these points is often considerable. The present instance points out the method of constructing the line DS, as distinctly as if it was of greater magnitude.

the sections; the volume immersed, by the inclination on the side ADW, being equal, in a practical sense, to the volume which emerges on the side BDN. Let each or either of these volumes be denoted by the letter A = 18509 cubic feet, being the average value between 18432 and 18587. Through the points D, in all the sections, draw lines F*f* perpendicular to the lines NW; the same plane passes through all the lines F*f*: in the next place, the distance between the centres of gravity of the volumes immersed, and caused to emerge, in consequence of the vessel's inclination, estimated in the direction of a line NW, perpendicular to the plane F*f*, is to be obtained. To effect this, the line D*l* *, in the principal section, is found by mensuration to be = 13.8, *l*U = 7.03; and the area † AW*b* = 2.95. The area ADW*b* has been found = 124.87. ‡ Wherefore, according to the preceding solution, the distance $lL = \frac{2.95 \times 7.03}{124.87} = 0.17$, which being added to the line D*l* = 13.8, the sum will be DL = 13.97, and the product arising from multiplying this line into the area ADW*b* will be $13.97 \times 124.87 = 1742$. Similar products being obtained, arising from multiplying the several areas ADW*b* into the perpendicular distances of their centres of gravity from the plane F*f*, also the several products arising from multiplying the areas BDN*c* into the perpendicular distances of their centres of gravity from the plane F*f*, in each of the 34 vertical sections, the results will be as expressed in the adjacent table.

* The point M is the centre of gravity of the triangle ADW, as in the general construction and solution: M*l* is drawn through M, perpendicular to NW: the line *l*L is found according to the method described in the same general solution.

† Page 288. ‡ Ibid.

Vertical Sections	Products on the Side ADH b	Products on the Side BDC c
1	359	143
2	929	562
3	1269	1010
4	1522	1342
5	1683	1542
6	1699	1669
7	1723	1766
8	1747	1853
9	1739	1873
10	1731	1892
11	1736	1912
12	1742	1931
13	1742	1931
14	1742	1931
15	1727	1930
16	1713	1929
17	1724	1915
18	1736	1901
19	1719	1866
20	1702	1832
21	1660	1798
22	1618	1764
23	1591	1705
24	1564	1646
25	1516	1584
26	1468	1522
27	1415	1434
28	1363	1346
29	1290	1220
30	1199	1088
31	1114	881
32	1010	673
33	919	415
34	708	102
	50119	49908

Suppose the volume immersed, by the inclination on the side ADW b, to be divided into very thin laminæ, or solids, the bases of which are the areas of the successive figures ADH b, and the thickness a small increment of the longer axis; by applying the Rule III. to the products in the adjacent table, corresponding to the figures ADW b, we shall obtain the sum of all the products, arising from multiplying each of these thin solids into the perpendicular distance of its centre of gravity from the plane $Ff = \overline{S+2P+3Q} \times \frac{15}{8} = 254367$; which sum of products being divided by the solid contents of the said volume, or 18509, will be the distance DQ, of the centre of gravity of the volume ADW b from the plane $Ff = \frac{254367}{18509} = 13.78$; and, by a similar calculation, the distance DP from the plane F f, of the centre of gravity of the volume on the side BDN c, caused to emerge by the inclination, will be $\frac{250627}{18509} = 13.54 = \text{DP}$.

The sum of these two lines, DQ + DP, or PQ, will be = 27.32 feet, which is the distance of the centres of gravity of the volumes immersed and emerged, in consequence of the vessel's inclination, esti-

mated in the direction of the line NW, perpendicular to the plane F*f*: let the line PQ be denoted by the letter *b*.

The solid contents of the entire volume displaced are next to be measured, from having first obtained the areas of the 12 horizontal sections, intersecting the vertical axis at the common interval of 2 feet, and dividing the immersed volume into 11 horizontal segments. The areas of the several horizontal sections are measured by Rule III. from having given the ordinates drawn in the said sections, parallel to the water's surface from head to stern, at the common interval of 5 feet.

The mensuration of the area of the horizontal section 12, which coincides with the water's surface, from the ordinates entered in the annexed * table, is as follows:

Vertical Sections	Ordinates of the Horizontal Section 12. Feet.	Vertical Sections	Ordinates of the Horizontal Section 12. Feet.
1	10.78	19	21.48
2	16.00	20	21.32
3	18.40	21	21.22
4	19.84	22	21.05
5	20.54	23	20.82
6	20.94	24	20.61
7	21.20	25	20.44
8	21.38	26	20.15
9	21.48	27	19.85
10	21.50	28	19.58
11	21.56	29	19.25
12	21.58	30	18.77
13	21.56	31	18.21
14	21.56	32	17.52
15	21.56	33	16.50
16	21.55	34	12.95
17	21.53		
18	21.51		674.19

According to the Rule III.

$$S = 23.73$$
$$P = 206.41$$
$$Q = 444.05$$

the area of half the horizontal section $12 = \overline{S + 2P + 3Q} \times \frac{15}{8} = 3316.3$, and the total area of the horizontal section $12 = 6633.6$ square feet.

The areas of the 12 horizontal sections, computed by Rule III. from the ordinates, as expressed in the table inserted in the Appendix, are as follows:

* See Table of Ordinates, in the Appendix.

Water-Lines.	Areas of the Horizontal Sections. Square Feet.
12	6633.6
11	6568.4
10	6499.3
9	6324.0
8	6238.0
7	5948.8
6	5687.8
5	5353.4
4	4906.6
3	4298.6
2	3417.0
1	925.0
	62800.5

The solid contents of the volume between the sections 12 and 2, is measured by the Rule II. by making the sum of the 12th and 2d areas $= S$ the sum of the 11th, 9th, 7th, 5th, and 3d, $= P$ the sum of all the other areas - $= Q$ the common interval between the areas being two feet.

The solid contents of the volume between the sections 12 and 2, is $\overline{S + 4P + 2Q} \times \frac{2}{3}$ $= 113791.2$ cubic feet.

The contents of the volume between the section 2 and 1, may be obtained by the Rules II. and III. For, by the Rule III. the contents between the sections 4 and 1, is found to be $= 21733.8$

By Rule II. the contents between the sections 4 and 2 is - - - - $=$ 17012.0

Contents between the sections 2 and 1 $=$ 4721.8

Contents between the sections 12 and 2 $=$ 113791.2

Contents between the sections 12 and 1 $=$ 118513.0

To the volume thus determined, the contents of the space between the horizontal section 1 and the keel, are to be added. To measure this volume, the areas must be first obtained of the 34 vertical sections which are included between the first ordinate in each vertical section and the keel; which areas are found, by the methods of mensuration already described, to be as in the annexed table.

Vertical Sections	Areas between the first Ordinate and the Keel. Square Feet.
1	
2	
3	
4	2.4
5	2.8
6	4.6
7	5.6
8	6.6
9	7.4
10	9.3
11	10.5
12	10.5
13	10.5
14	10.5
15	10.5
16	10.5
17	9.8
18	9.2
19	8.5
20	8.0
21	6.0
22	5.3
23	4.8
24	4.3
25	3.7
26	3.0
27	2.4
28	1.9
29	1.6
30	1.4
31	1.2
32	1.1
33	1.0
34	1.0
	175.9

Applying the Rule III. to these areas, and making the common distance between them $= r = 5$ feet, the solid contents between the first horizontal section and the keel is found to be $\overline{S + 2P + 3Q} \times \frac{3r}{8} = 871.0$ feet.

	Feet.
Contents between the horizontal sections 12 and 1 - -	118513.0
Contents between the first section at the keel* - - -	871.0
Total contents of the volume immersed between the first and thirty-fourth vertical section - - -	119384.0

Let this volume be represented by the letter V.

By the preceding computations, the line PQ $= b$ was found $= 27.32$, and the volume immersed by the vessel's inclination, or $A = 118509$. From these determinations, the line ET is inferred; for, according to the general theorem,

as V : A :: b : ET or
119384 18509 27.32 4.23
wherefore $\text{ET} = 4.23$.

* The rules here employed for measuring the volume displaced, cannot be applied to the irregular parts of the vessel, adjacent to the head and stern, without constructing and measuring the ordinates by which their forms are defined: the same observation applies to the mensuration of the body of the keel and of the rudder. But, as these additional volumes bear a small proportion to the entire volume displaced, they may be determined by an estimate founded on the draught of the vessel, sufficiently near the truth, without taking the trouble of a more rigorous calculation by equidistant ordinates.

To infer the measure of stability from this value of the line ET, it will be necessary to have given the distance GE, between the centre of gravity of the vessel, and the centre of gravity of the volume of water displaced. The position of this latter centre is regulated entirely by the form and dimensions of the body under water; and, on this account, is to be considered as a point absolutely fixed, in respect of the water-section, or other given plane. But the position of the vessel's centre of gravity being regulated, partly by the construction and equipment of the vessel, and partly by the distribution of the lading and ballast, can be assumed on the ground of supposition only; unless in cases where the position of this point has been actually ascertained. In some vessels, the distance GE has been measured, and found equal to about $\frac{1}{8}$ part of the greatest breadth at the water-line: without knowing what the real distance of the two centres of gravity G and E may be, in the ship of which the dimensions are here given as subjects of calculation, the distance GE may be estimated (merely for the purpose of exemplifying the preceding rules) at $\frac{1}{8}$ of the breadth BA, or $\frac{43.16}{8} = 5.39$. Consequently, the inclination of the vessel from the upright being $30°$, $GE \times \sin. 30° = \frac{5.39}{2} = 2.69 = ER$: which being subtracted from $ET = 4.23$, will leave TR or $GZ = 1.53$ feet, the measure of the vessel's stability, when inclined round the longer axis through an angle of 30°.

The solid contents of the volume displaced being 119384 cubic feet, the weight of the vessel and contents will be equal to that of 119384 cubic feet of sea water; which, allowing 35 cubic feet to each ton, will amount to 3410 tons. According to this determination, the force of stability to turn the vessel round the longer axis, when inclined from the upright through an angle of 30°, is a force of 3410 tons, acting at a distance of 1.53 feet

from the axis; which force is equivalent to a weight or pressure* of 241 tons, acting at a distance of 21.58, or half the breadth at the water-line from the axis.

In this computation, the distance GE, between the centres of gravity G and E, has been assumed without considering the absolute position of these points, in respect to the water-section or keel. But the distance DE, or OE, ought to be known, since the point E being fixed in the same vessel, when the weight is given, and the centre of gravity G being within certain limits moveable, the adjustment of this centre, by means of the lading and ballast, will be better regulated, if the position of the point E be first ascertained: the distance DE will be found from having given the areas of the 12 horizontal sections, and the contents of the volume between the section 1 and the keel.

Suppose the areas of 12 horizontal sections of the vessel to be given, as they are expressed in page 294; let the displaced volume be conceived divided into laminæ, or very thin solids, of which the bases are the areas of the successive horizontal sections, and the thickness a small increment of the vertical axis. The sum of the products arising from multiplying each of these segments into its perpendicular distance from the section 12, also a similar sum of products for the solid contents adjacent to the keel, will be found from the computation subjoined.

* This equivalent weight is not here supposed to be counterbalanced by the wind, which probably never acts with sufficient force to keep a vessel of this weight inclined permanently from the upright to so great an angle. But the measure of force which acts to turn the vessel round the longer axis, when inclined to this angle of 30°, is precisely that which is here stated, according to the given conditions.

Water-lines, or horizontal Sections.	Areas of the horizontal Sections. Square feet.	Distances from the Section 12.	Products of each area, multiplied into its distance from the Section 12.	Numbers for computing according to the Rule II.	Products.
12	6633.6	0	00000.0	1	
11	6568.4	2	13136.8	4	52547.2
10	6499.3	4	25997.2	2	51994.4
9	6324.0	6	37944.0	4	151776.0
8	6238.0	8	49904.0	2	99808.
7	5948.8	10	59488.0	4	237952.
6	5687.8	12	68253.6	2	136507.2
5	5353.4	14	74947.6	4	299790.4
4	4906.6	16	78505.6	1	78505.6
				Sum of the products =	1108880.8

The common interval between the sections being 2 feet, the sum of all the products arising from multiplying each thin horizontal segment contained between the section 12 and 4, into its distance from the water-section, will be $\frac{2}{3}$ of 1108880.8 = 739253, by the Rule II.

				Numbers for computing according to Rule III.	
4	4906.6	16	78505.6	1	78505.6
3	4298.6	18	77374.8	3	232124.4
2	3417.0	20	68340.0	3	205020.0
1	925.0	22	20350.0	1	20350.0
					536000.0

Sum of the products arising from multiplying each thin horizontal segment between the section 4 and 1, into its distance

from the section 12, $= \frac{3}{4} \times 536000$, by the Rule III. = 402000
Sum of the products between section 12 and 4 = 739253
*The contents of the volume between the keel and the ordinate 1, multiplied into the distance of its centre of gravity from the section 12 - = 19465
Sum of the products arising from multiplying each horizontal evanescent solid into its distance from the water-section - - - = 1160718

The sum of products, thus found, divided by the entire volume displaced, will be the distance of the centre of gravity of that volume from the water-section, or

$$DE = \frac{1160718}{119384} = 9.7224 \text{ feet.}$$

If the distance between the centres of gravity G and E should be assumed $\frac{1}{8}$ of the breadth BA at the water-line, or 5.39 feet, this distance being subtracted from 9.22 feet, will be the distance of the vessel's centre of gravity beneath the water-section, or DG = 3.83 feet.

To determine the limiting point or metacentre W, above which if the centre of gravity G should be raised the vessel will overset, it is only necessary to compute, by means of the Rule II. or III. the value of the line $EW = \frac{\text{Fluent of } BA^3 \times \dot{z}}{12V}$: where z represents any portion of the longer axis: AB = the breadth of the water-section, at the distance z from the initial point where the mensuration commences: $\dot{z}$ = a small increment of z: V = the solid contents of the volume displaced.

By computing the cubes of each ordinate of the water-section, drawn at the common interval of 5 feet, as represented in the

* Equal to the sum of the products arising from multiplying each thin horizontal segment between the section 1 and the keel, into its distance from the water-section.

table* of ordinates, the total sum of these cubes is = 276573.21

Cube of the 1st ordinate, or 10.78 = 1252.73

Cube of the 34th ordinate, or 12.95 = 2171.75

Sum S = 3424.48

Sum of the cubes of the 4th, 7th, 10th, &c. 28th, and 31st = P = 88628.41

S + P = 92052.89

Sum of the cubes of the 2d, 3d, 5th, 6th, &c. 32d, 33d = Q - - - - = 184520.32

$S + 2P + 3Q = 734242.26$, this sum $\times 15$, will be = fluent of $BA^3 \times \dot{z} = 11{,}013{,}633.9$; and since $V = 119384.0$, $EW = \frac{11{,}013{,}633.9}{12 \times 119384.} = 7.688$ feet.

In this vessel, the centre of the immersed volume E is 9.72 feet beneath the water's surface; it follows, that the metacentre will be 2.03 feet beneath the water's surface.

The total weight of a vessel and contents is inferred from knowing the volume of water displaced by the vessel, the solid contents of which space have been calculated in the preceding pages, from the areas of the 12 horizontal sections intersecting the vertical axis at a common interval of 2 feet. By similar calculations, we may determine the several weights of tonnage which will cause the vessel to sink to any different depths, estimated by the horizontal line or section which is coincident with the water's surface.

The solid contents of the volume included between the horizontal sections 12 and 9, is found, by the Rule III. to be 39120 cubic feet; displacing a weight of water (allowing 35 feet to

* See Appendix.

a ton) of 1117.6 tons: the volume between the sections 11 and 9, is found, by the Rule II. to be = 25926, displacing 741 tons of water: the volume between the sections 12 and 11 will therefore displace a weight = 377 tons.

By similar computations, the following results are obtained.

From the water-section.	Difference of tonnage.	From	Difference of tonnage.
12 to 11	377 tons.	12 to 11	377 tons.
11 to 10	374	12 to 10	751
10 to 9	367	12 to 9	1118
9 to 8	357	12 to 8	1475
8 to 7	348	12 to 7	1823
7 to 6	333	12 to 6	2156

If any one of the adjustments determining the stability of a vessel should be altered, the several other conditions on which that power depends, will most commonly experience corresponding changes, the effects of which it is not easy to estimate, without some reference to the theory of stability. If the weight of the Cuffnells and contents should be diminished 751 tons; or rather if another vessel, constructed in all respects like the Cuffnells, should be loaded by 751 tons less weight, the following changes will take place, by which the stability is principally affected; one of which is additive to, and the others subtractive from, the stability of the vessel.

1st. The section of the water will be nearer to the keel by 4 feet; so to coincide with the horizontal section 10 instead of 12.

The centre of gravity of the displaced volume will be nearer to the keel than before by - - 2.19 feet.

Admitting, therefore, in the first instance, the

centre of gravity* of the vessel and contents to remain at the same distance from the keel, the distance of that centre from the centre of the displaced volume will be increased by 2.19 feet
and this change will operate to diminish the stability.

2dly. The total weight being less, will have the effect of diminishing the stability, in the proportion of - - - 3410 to 2660

3dly. The breadths of the vessel at the water-line are less than before, being the double ordinates in the horizontal† section 10, instead of those in the horizontal section 12. By this change, the stability will also be diminished.

4thly. The volume displaced being less, in consequence of the diminished tonnage, in the proportion of 2660 to 3410, the stability will be augmented by this change; not in the proportion of 2660 to 3410, but in a proportion considerably greater; yet not sufficient to counterbalance the effect of the alterations by which the stability is diminished: on the

* Let the distance of the centre of gravity of any body, or system of bodies, from a given plane, be denoted by the letter D: if weights are taken away from different parts of the system, in such a proportion that the sum of the products arising from multiplying each removed weight into its distance from the given plane, divided by the sum of the removed weights, may be equal to the distance D, the distance of the centre of gravity of the remaining weights, from the same plane, will also be the distance D: that is, the distance of the common centre of the whole system, from the given plane, will not be affected by the removal of any portion of the weights, according to the conditions here described.

† See Appendix.

whole, these alterations will diminish the stability of the vessel, when inclined to a given small angle, in the proportion of about* - 100 to 67

In this estimate, the vessel's centre of gravity has been supposed to remain at the same distance from the keel at which it was situated in the Cuffnells, and consequently more remote from the centre of the displaced volume by 2.19 feet: suppose the vessel's centre of gravity to be depressed 2.19 feet, by altering the distribution of the lading and ballast, so as to be at the same distance from the centre of the displaced volume, as in the Cuffnells; the effect of this alteration will be entirely additive to the stability, which will be increased in the proportion of 47 to 100.

An increase in the proportion of 47 to 100, combined with a diminution in the proportion of 100 to 67, is, on the whole, an increase of stability, in the proportion of 47 to 67, or about† 7 to 10

It might be difficult to depress the vessel's centre of gravity

* The subject of these observations being the relative stabilities of the two vessels, they are supposed to be inclined from the upright to the same angle, which may be assumed of any magnitude, either great or small: the latter supposition is here adopted, which is well suited to the purpose of general illustration. But nothing can be inferred from these results, respecting the stabilities, when the angles of inclination are considerable; which are to be obtained from computations founded on the methods which have been described in the preceding pages.

† This proportion might have been immediately inferred from one computation only; but, by calculating the effects of diminishing the vessel's weight separately, the increase and diminution of stability, arising from the alteration of the several conditions, are more distinctly expressed.

through so great a space as 2.19 feet; and the increase of stability which would ensue from it, may perhaps not be necessary. If it should be required that the stability of the vessel, when the weight is diminished 751 tons, shall be just equal to that of the Cuffnells, this will be effected by adjusting the centre of gravity lower than its original position, by only .97 parts of a foot.

These determinations relate to the vessel's stability in respect to the longer axis. But the position of the shorter axis, round which the ship revolves in pitching, and of the vertical axis, round which it is caused to turn by any horizontal force not passing through the vertical axis, will also experience some change in consequence of diminishing the vessel's weight. For the centre of gravity of the volume displaced, being necessarily in the same vertical line with the vessel's centre of gravity when it floats quiescent, fixes the position of the latter point, in respect to the ship's length, when floating on an even keel. And since the alteration of the water-section, of 4 feet in height, causes the centre of gravity of the displaced volume to approach nearer to the head of the vessel by about $\frac{1}{2}$ a foot, both the shorter horizontal axis and vertical axis of the vessel must experience the same change of position: the former alteration affects the motion of the vessel in pitching, and the latter somewhat increases the action of the rudder in turning the ship, and also affects the motion of the vessel, in turning to and from the wind, by causes independent of the rudder.*

These observations point out the alterations of stability, in

* By altering the distance of the centre of gravity from the points of application, in the longer axis, at which the water's resistance and force of the wind, when not exactly balanced, act on the vessel.

consequence of diminishing the tonnage of the vessel, without entering into any consideration how far such changes are, on the whole, beneficial or otherwise.

It is here necessary to observe, that the force of stability and the measure of it, the subject of investigation in the preceding pages, is wholly independent of the water's resistance, which co-operates with the vessel's stability only while it is inclining, and wholly ceases as soon as the vessel has attained to the greatest inclination, at which it is supposed permanently to remain in a state of equilibrium; the inclining force being exactly balanced by the force of stability. This observation will obviate any difficulty that might possibly occur from the principle stated in page 213; which is, that if the shape of the zone WHFC, (fig. 1 and 2.) comprehending that portion of the sides of a vessel which may be immersed under, and may emerge above, the water's surface, should be the same in two vessels, the stability will be the same at all equal angles from the upright, whatever shape be given to the form of the volume immersed, which is situated beneath the said zone, provided the vessels be in other respects similarly constructed and adjusted: if, for instance, the keel of one vessel should be very deep under the body of the vessel, the keel of the other being of the ordinary dimensions, the deeper keel will oppose an increased resistance to the inclination of the vessel only while it is inclining, so as make it heel slower; but will not alter the angle of permanent inclination caused by a given force of the wind, or other uniform power; which inclination depends entirely on the stability, which has been determined in the preceding pages, and has no relation to the resist-

ance of the water, which arises from the vessel's motion round its longer axis.

The object of the preceding propositions, and inferences founded on them, has been rather to establish general principles, which may be of use in forming plans of construction, than to investigate what modes of construction are the most advantageous; a discussion more extensive than would be consistent with the subject here proposed to be considered, which relates to the stability of vessels only.

The practice of navigation requires the co-operation of many qualities in vessels, the laws and powers of which, considered as acting either separately or conjointly, it is the employment of theory to investigate. In respect to the construction of ships, it is obvious that no one of the component qualities can be regulated, without paying attention to all the others; because, by increasing or diminishing any of the powers of action, the others are commonly more or less influenced. It has been shewn, by the propositions demonstrated in these pages, that there are many practical methods by which the stability of vessels, at any given angle from the upright, may be augmented; a circumstance which gives to the constructor great choice of means for regulating this power, according to the particular service for which the ship is designed; for it is not every mode that will be advantageous. The several varieties of form and adjustment by which stability is increased, may be so unskilfully combined, that, in consequence of the very means used to obtain that essential quality, either the ship shall not steer well, or shall drift too much to leeward, or shall be liable to sudden and irregular motions in rolling, by which

the masts are endangered; or those angular oscillations of the ship shall be performed round an axis situated so much beneath the water's surface, that the motion of rolling shall be excessive and laborious. It is the proper use of theory, or right principle, whencesoever derived, so to adapt the means to the end proposed, that the required stability shall be imparted, without producing inconveniences of any kind, or such only as are unavoidable, and are the least prejudicial: the same observation applies to the other qualities of vessels. By duly combining the whole, ships are constructed so as to fulfil the purposes of navigation.

APPENDIX.

Note to the Investigation, Pages 232, 233.

$$\sqrt{\text{sine O} \times \text{sine P}} = \frac{\sqrt{1 - \text{tang.}^2 \tfrac{1}{2}\text{F} \times \text{tang.}^2 \text{S}}}{\text{sec.} \tfrac{1}{2}\text{F} \times \text{sec. S}}$$

This is investigated in the following manner:

Let R be put to denote a right angle, the other notation being the same as in page 232;

Then the angle $\text{DWP} = \frac{1}{2}\text{F} + \text{P}$; also $\text{DWP} = \text{R} - \text{S}$ wherefore $\frac{1}{2}\text{F} + \text{P} = \text{R} - \text{S}$ and $\text{P} = \text{R} - \frac{1}{2}\text{F} - \text{S}$, and, since $\text{O} = 2\text{R} - \text{F} - \text{P}$, it follows that $\text{O} = \text{R} - \frac{1}{2}\text{F} + \text{S}$;

consequently $\text{sin. P} = \text{cos.} \overline{\tfrac{1}{2}\text{F} + \text{S}}$

$\text{sin. O} = \text{cos.} \overline{\tfrac{1}{2}\text{F} - \text{S}}$

and $\text{sin. O} \times \text{sin. P} = \text{cos.} \overline{\tfrac{1}{2}\text{F} + \text{S}} \times \text{cos.} \overline{\tfrac{1}{2}\text{F} - \text{S}}$

$= \text{cos.}^2 \tfrac{1}{2}\text{F} \times \text{cos.}^2 \text{S} - \text{sin.}^2 \tfrac{1}{2}\text{F} \times \text{sin.}^2 \text{S}$

or because $\text{cos.}^2 \text{S} = 1 - \text{sin.}^2 \text{S}$

$\text{sin. O} \times \text{sin. P} = \text{cos.}^2 \tfrac{1}{2}\text{F} \times \overline{1 - \text{sin.}^2 \text{S}} - \text{sin.}^2 \tfrac{1}{2}\text{F} \times \text{sin.}^2 \text{S}$

or $\text{sin. O} \times \text{sin. P} = \text{cos.}^2 \tfrac{1}{2}\text{F} - \text{sin.}^2 \text{S}$

$$= \frac{\text{sec.}^2 \tfrac{1}{2}\text{F} \times \text{sec.}^2 \text{S}}{\text{sec.}^2 \tfrac{1}{2}\text{F} \times \text{sec.}^2 \text{S}} \times \overline{\text{cos.}^2 \tfrac{1}{2}\text{F} - \text{sin.}^2 \text{S}}$$

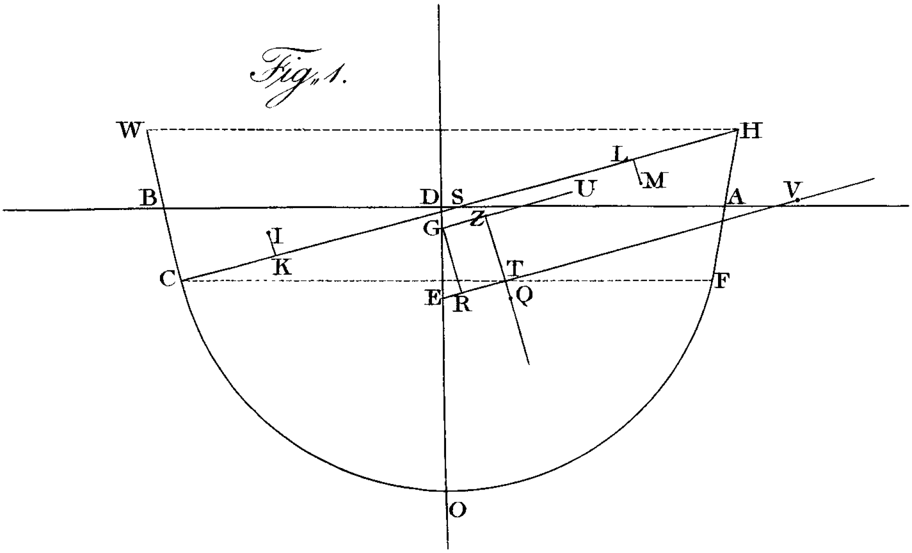
or $$\text{sin. O} \times \text{sin. P} = \frac{\text{sec.}^2 \text{S} - \text{sec.}^2 \tfrac{1}{2}\text{F} \times \text{tang.}^2 \text{S}}{\text{sec.}^2 \tfrac{1}{2}\text{F} \times \text{sec.}^2 \text{S}}$$

$$= \frac{1 + \text{tang.}^2 \text{S} - \overline{1 + \text{tang.}^2 \tfrac{1}{2}\text{F}} \times \text{tang.}^2 \text{S}}{\text{sec.}^2 \tfrac{1}{2}\text{F} \times \text{sec.}^2 \text{S}}$$

or $$\text{sin. O} \times \text{sin. P} = \frac{1 - \text{tang.}^2 \tfrac{1}{2}\text{F} \times \text{tang.}^2 \text{S}}{\text{sec.}^2 \tfrac{1}{2}\text{F} \times \text{sec.}^2 \text{S}}$$

Finally, $$\sqrt{\text{sin. O} \times \text{sin. P}} = \frac{\sqrt{1 - \text{tang.}^2 \tfrac{1}{2}\text{F} \times \text{tang.}^2 \text{S}}}{\text{sec.} \tfrac{1}{2}\text{F} \times \text{sec. S}}.$$

Fig. 1.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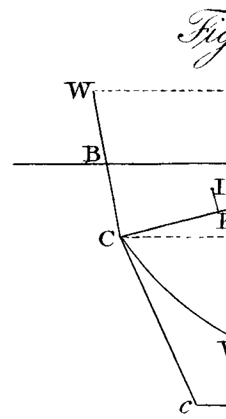
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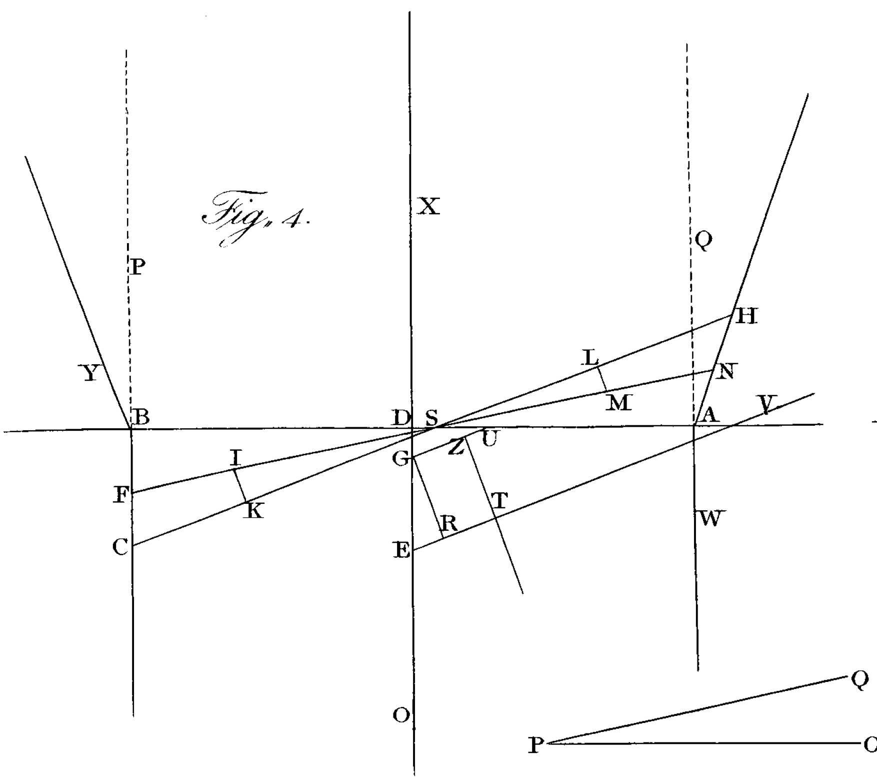
Fig. 4.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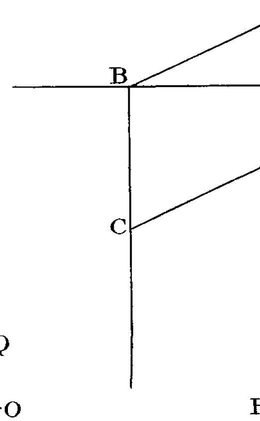
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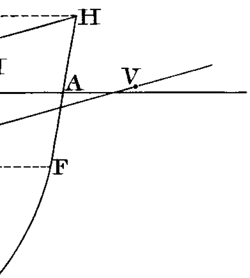
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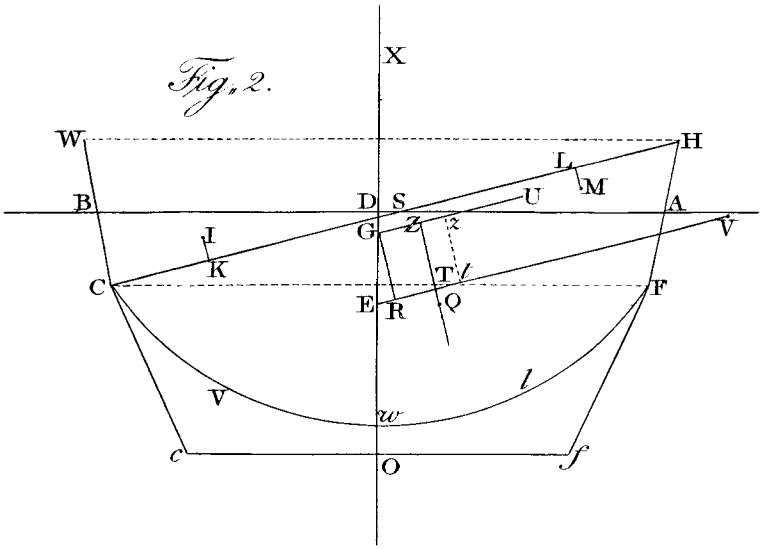
Fig. 2.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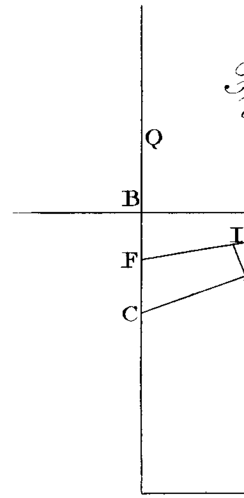
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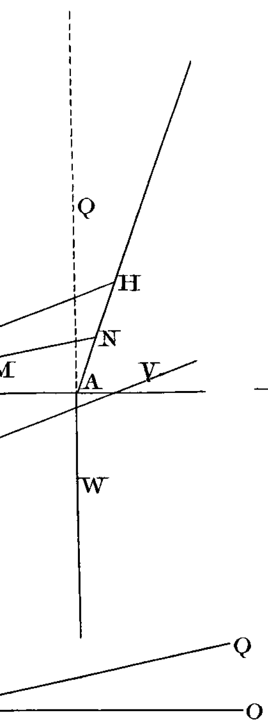
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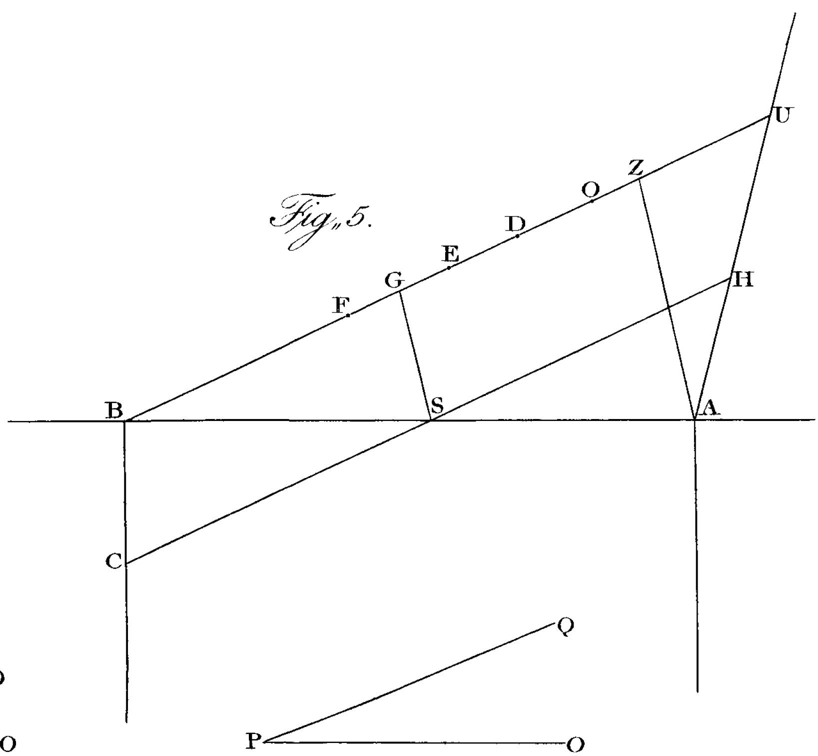
Fig. 5.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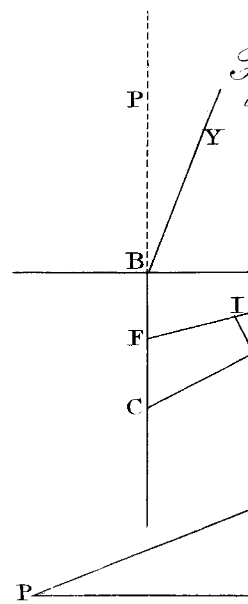
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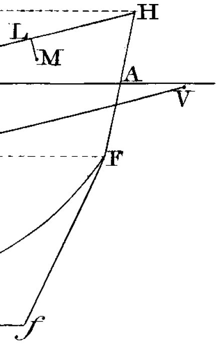
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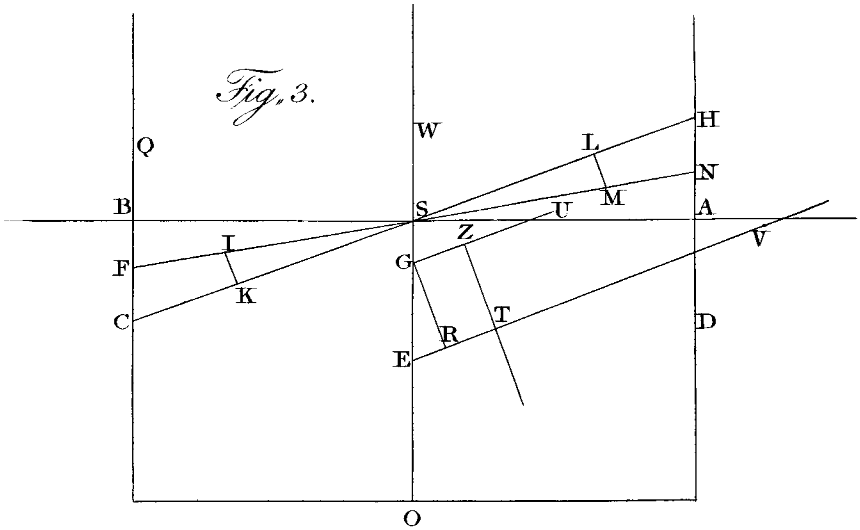
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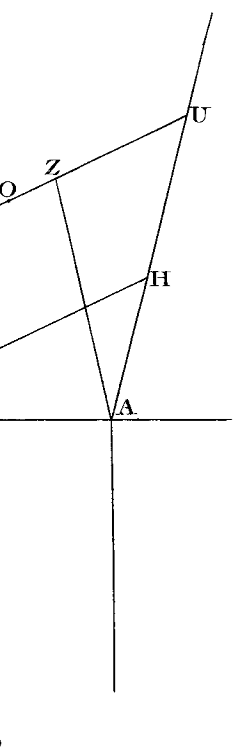
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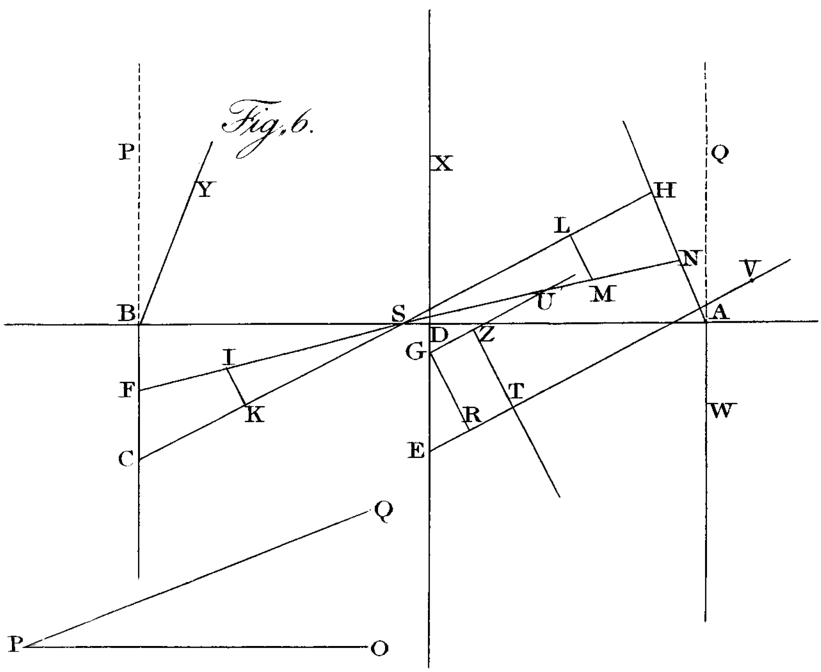
Fig. 6.
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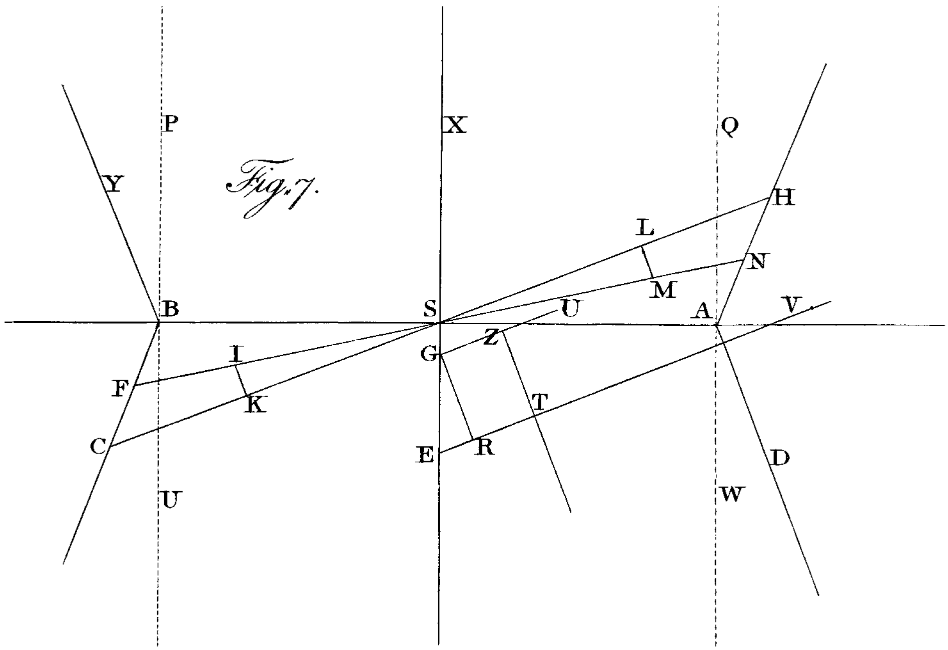
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Fig. 7.

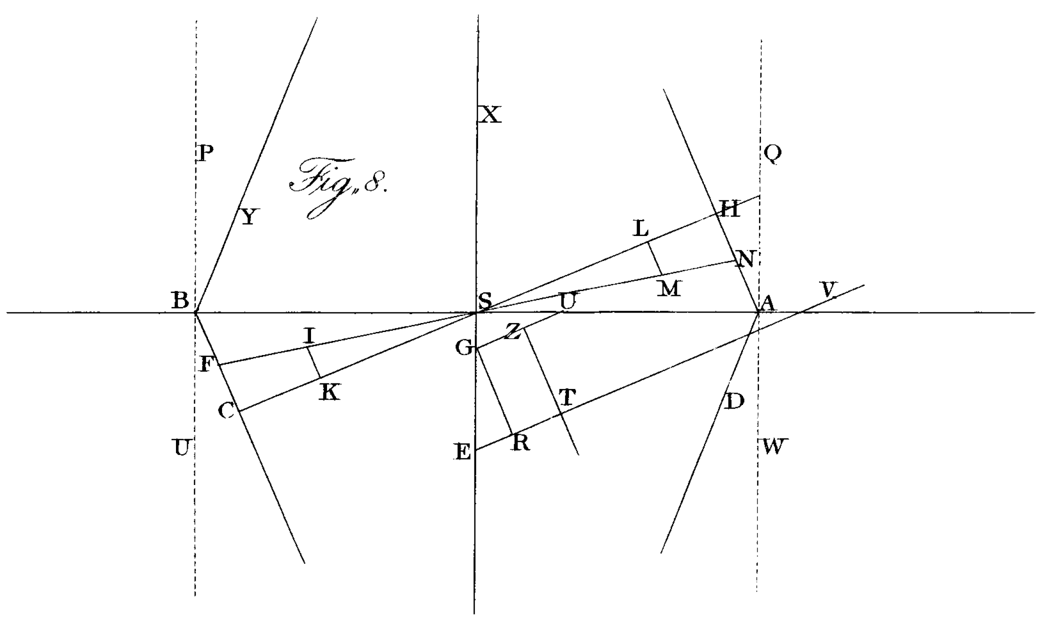
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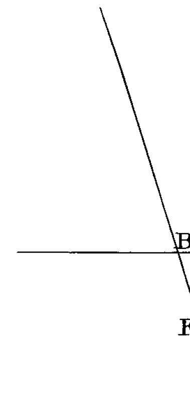
Fig. 8.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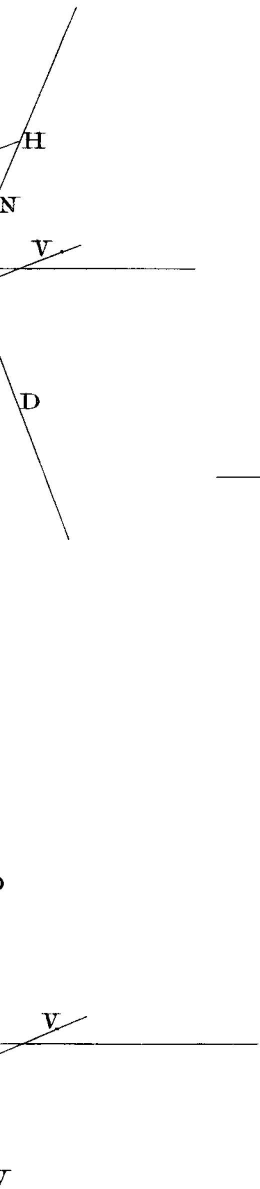


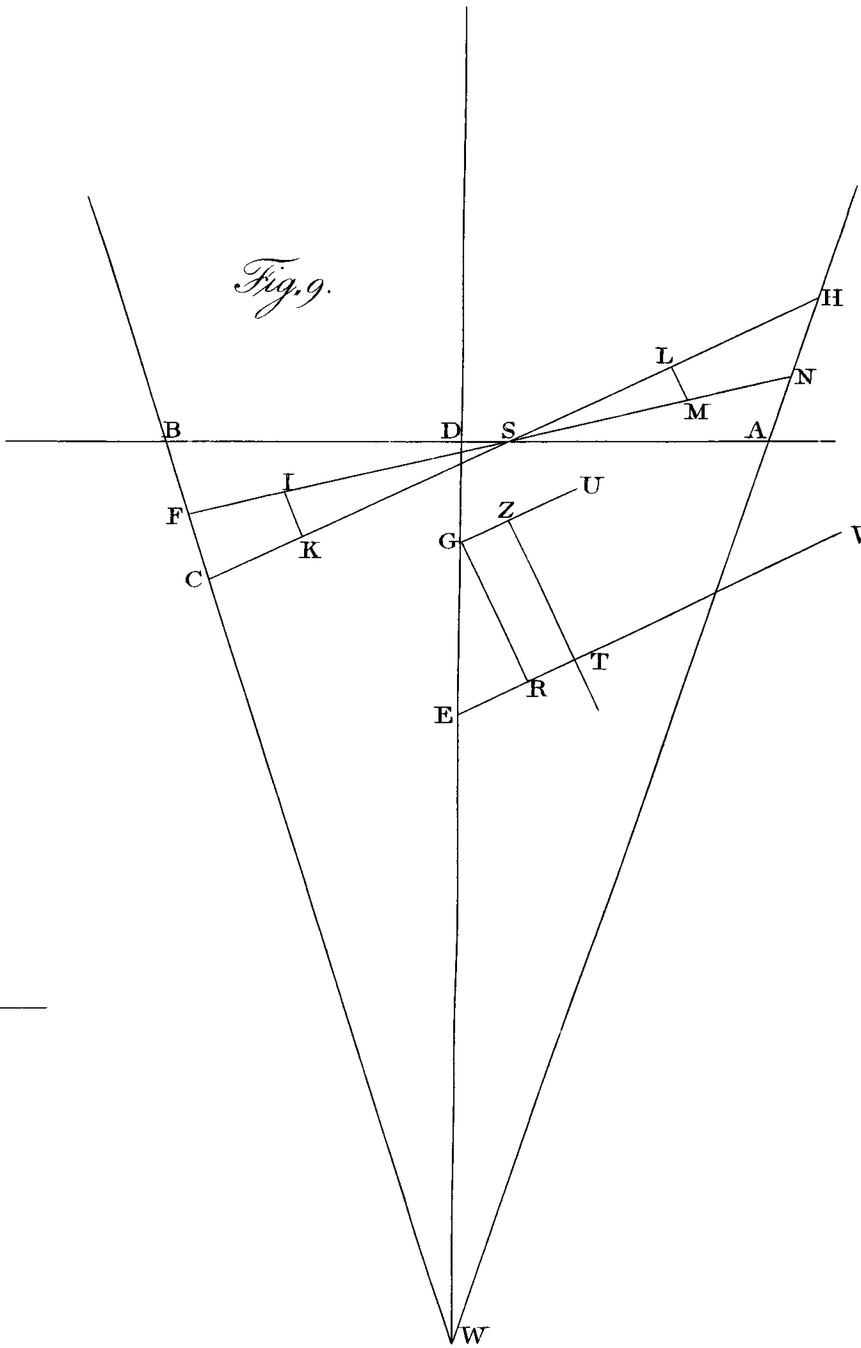
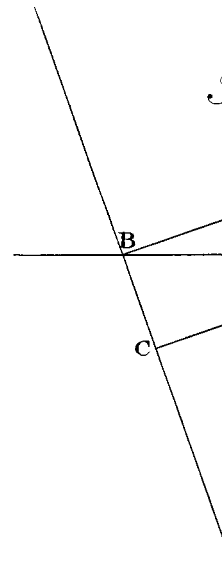


Fig. 9.



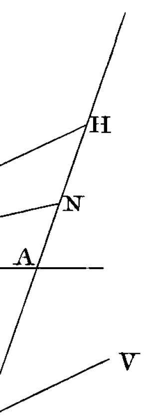


Fig. 10.

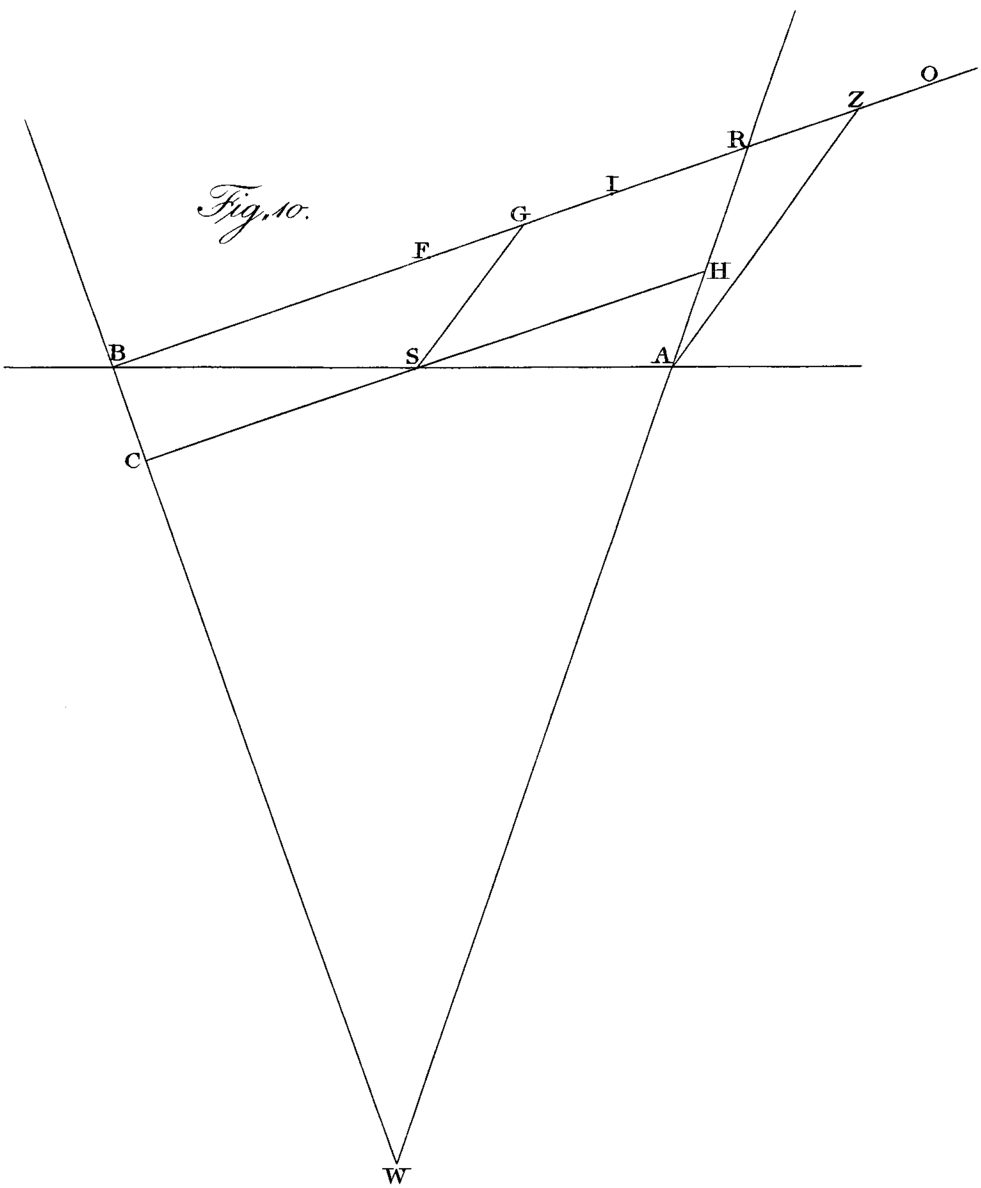
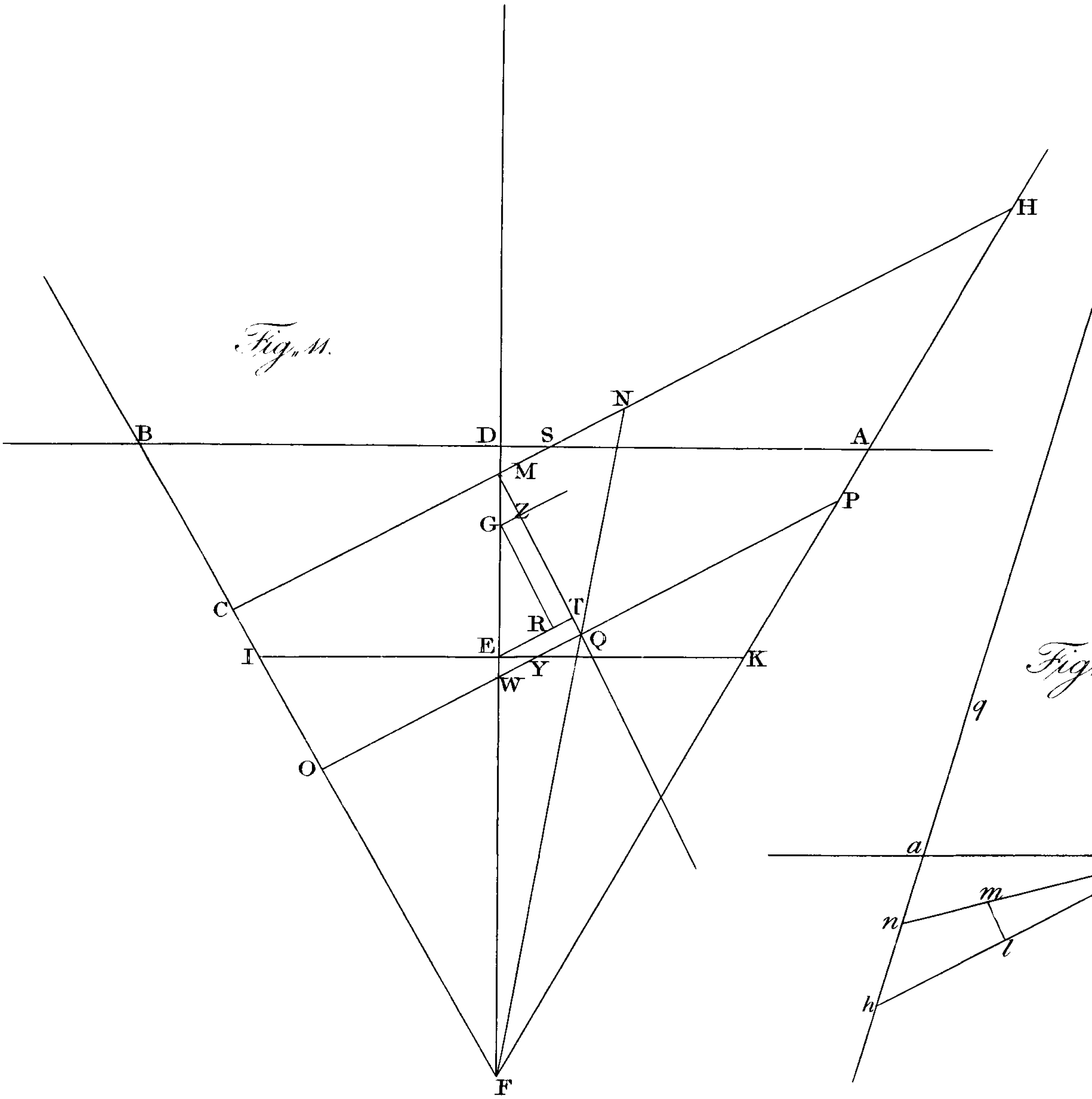


Fig. 11.

Fig.



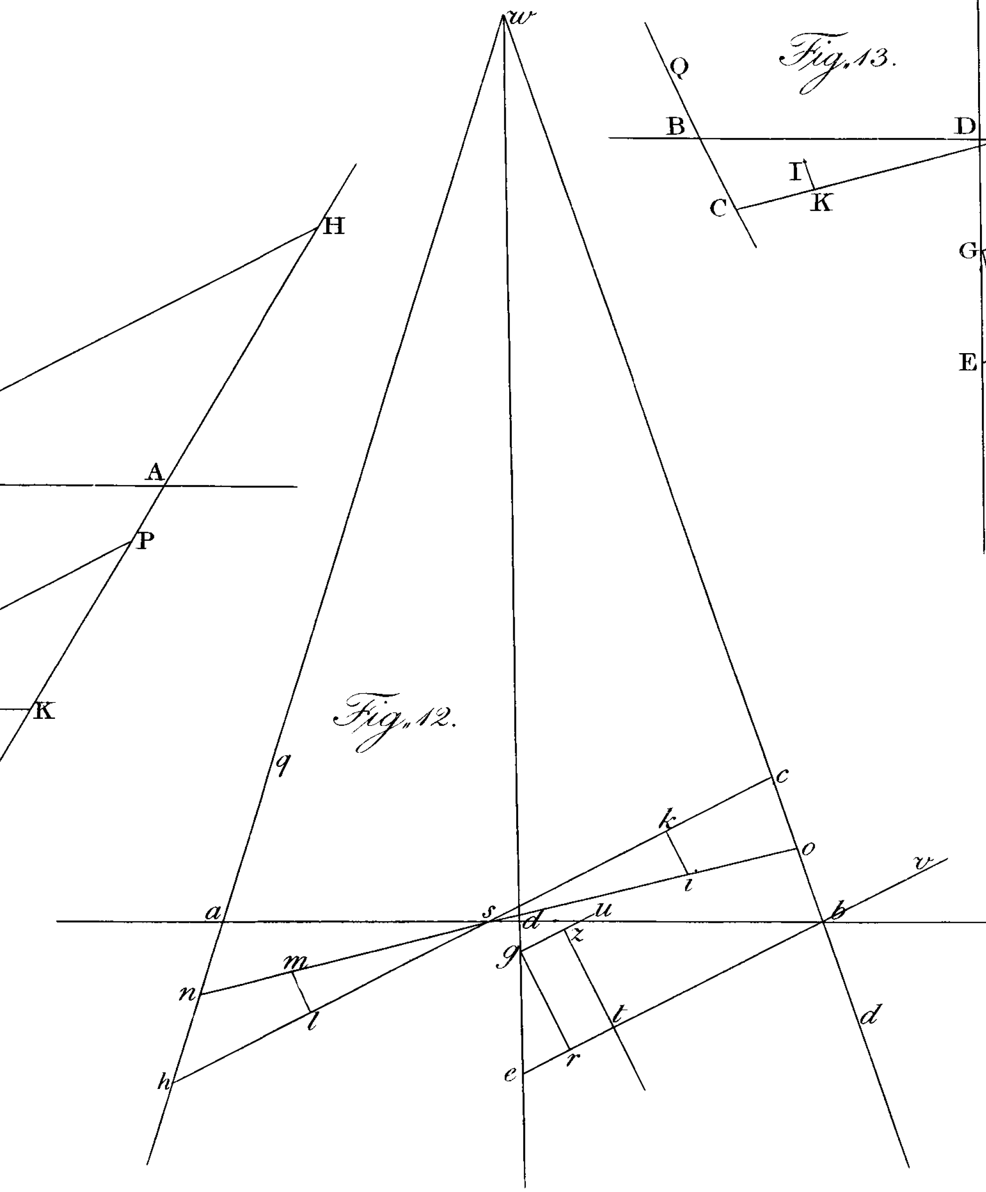


Fig. 12.

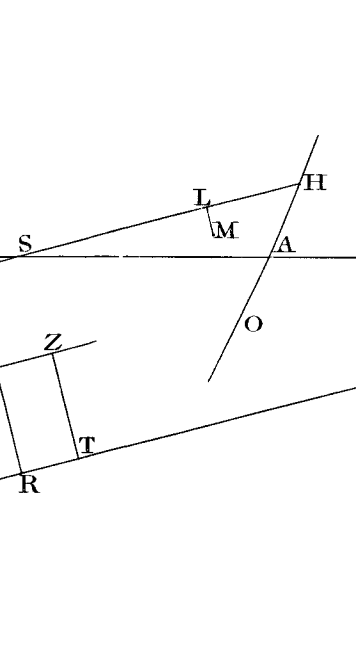
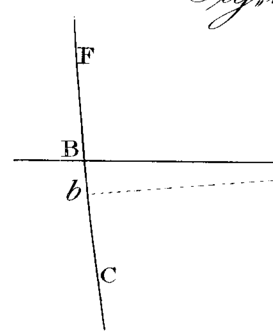


Fig. 13.



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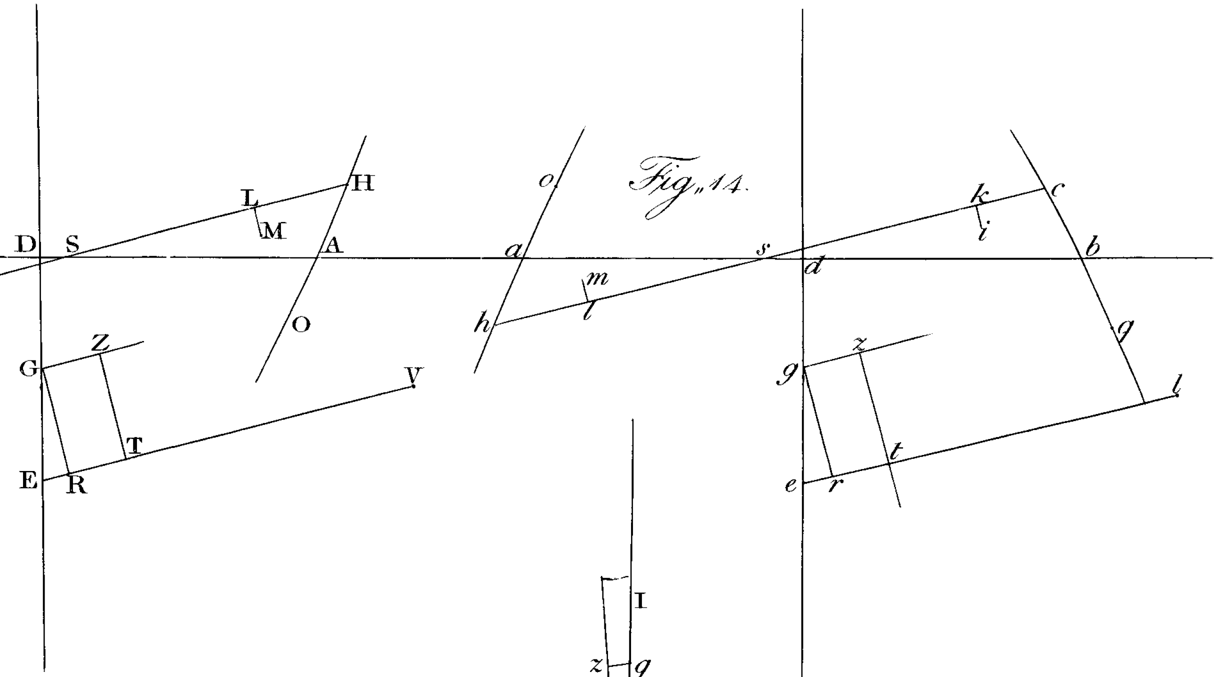

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Fig. 14.

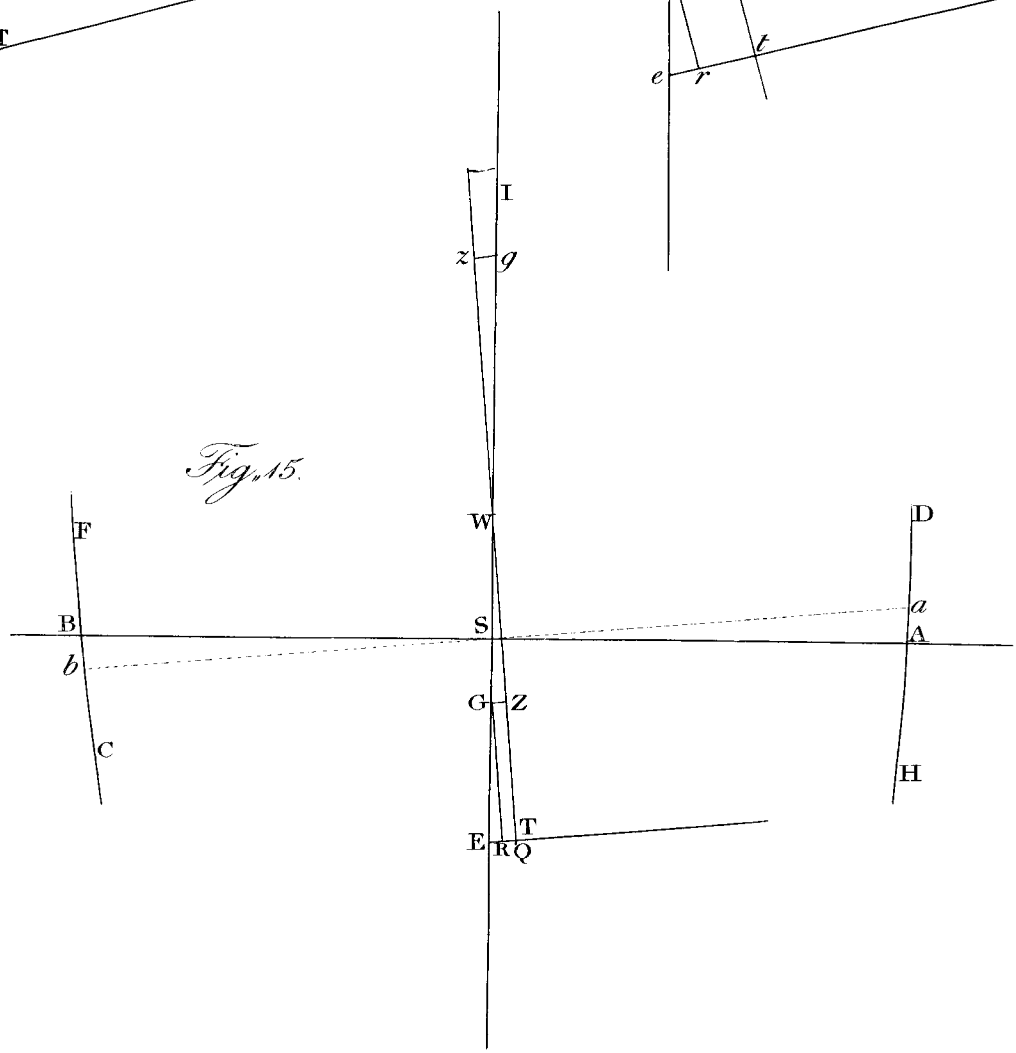

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Fig. 15.

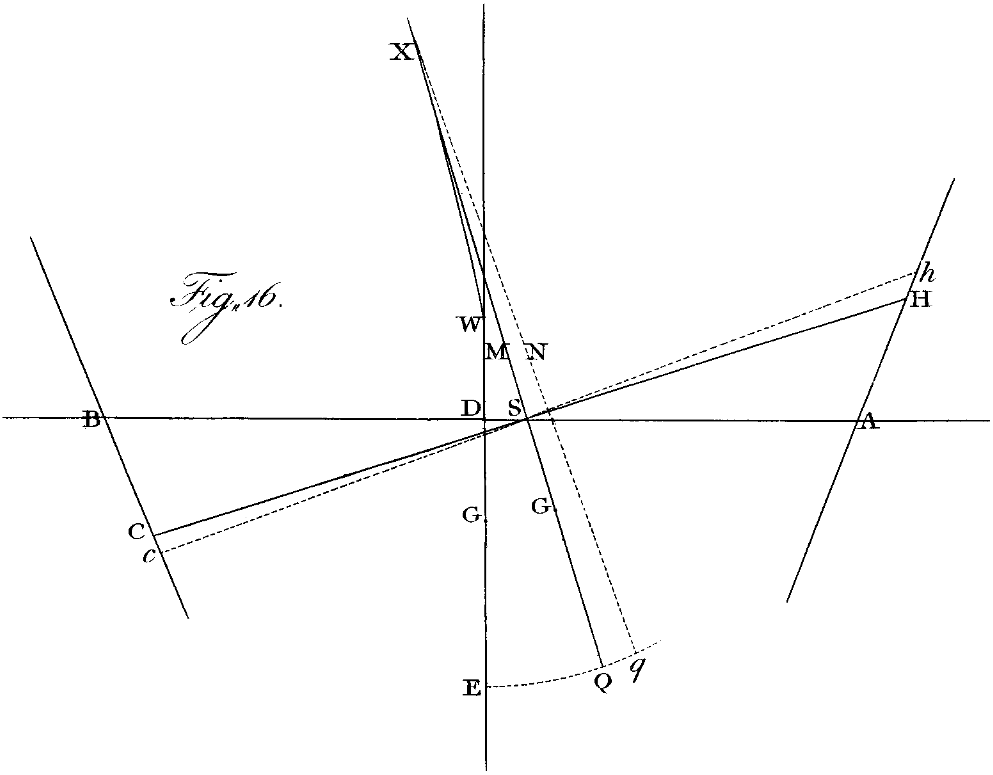


Fig. 16.

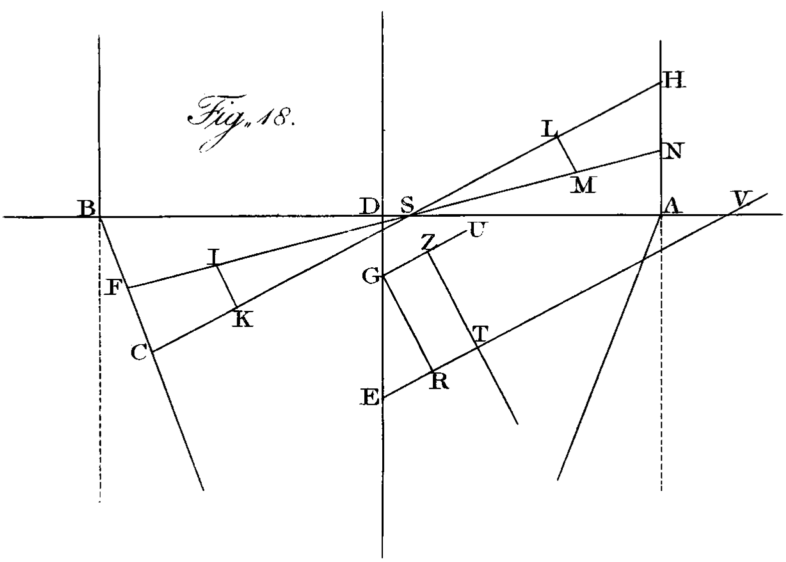


Fig. 18.

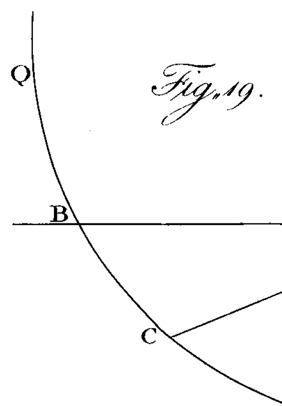
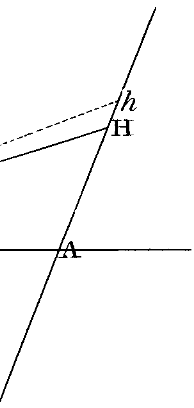


Fig. 19.



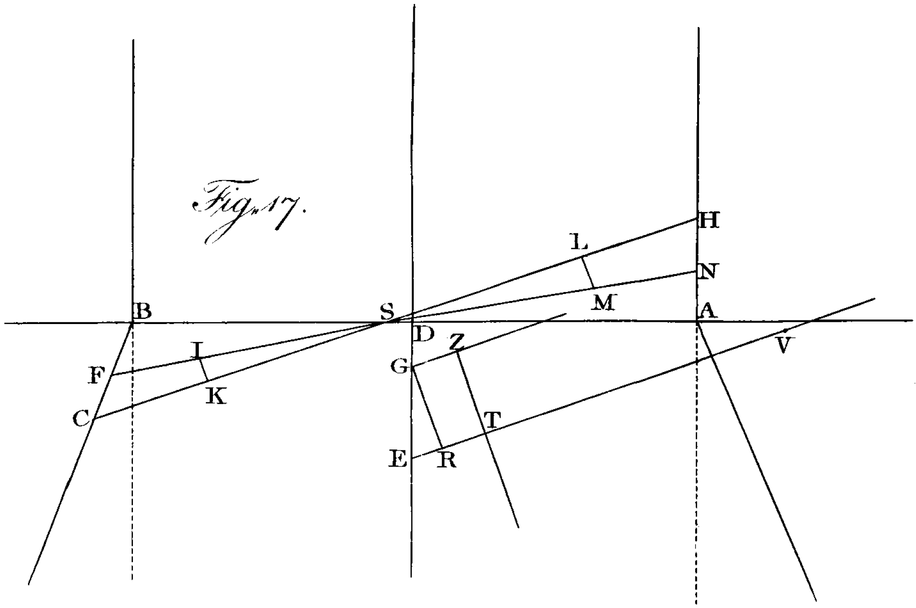
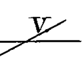


Fig. 17.



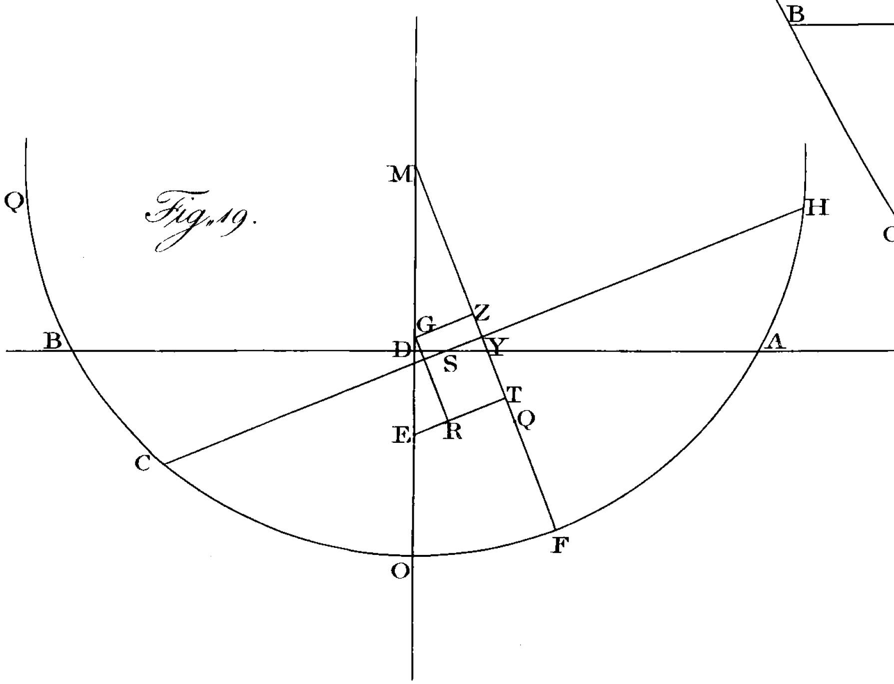


Fig. 19.

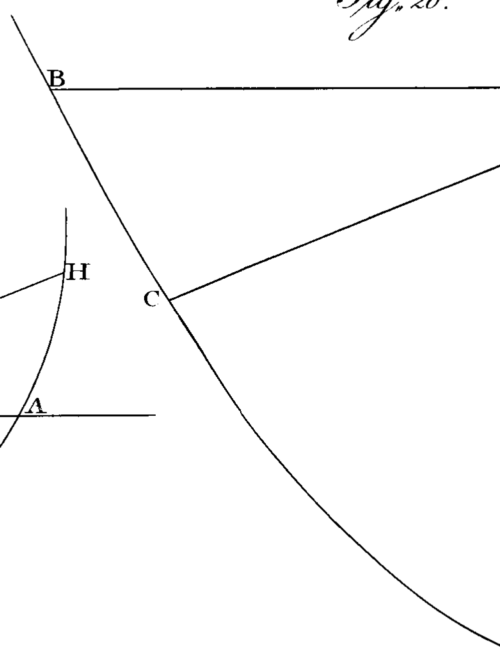


Fig. 20.

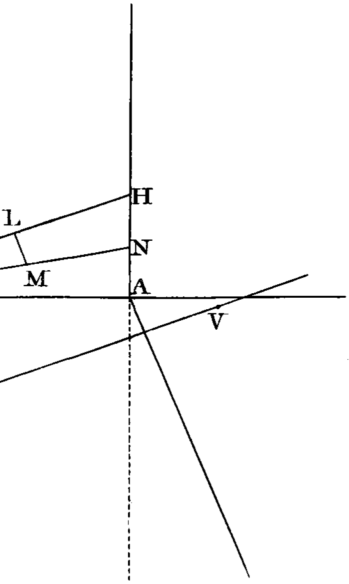
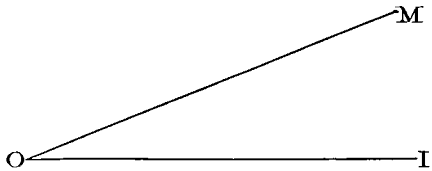
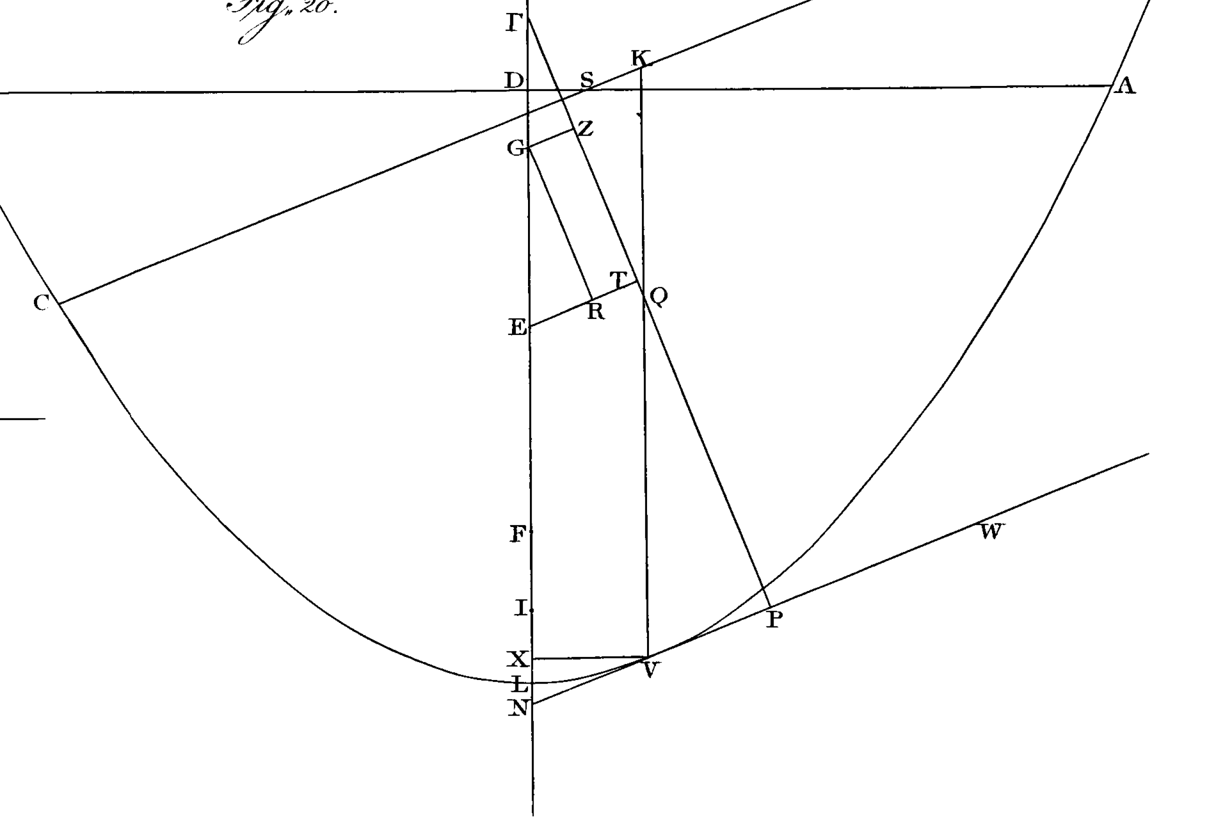


Fig. 20.





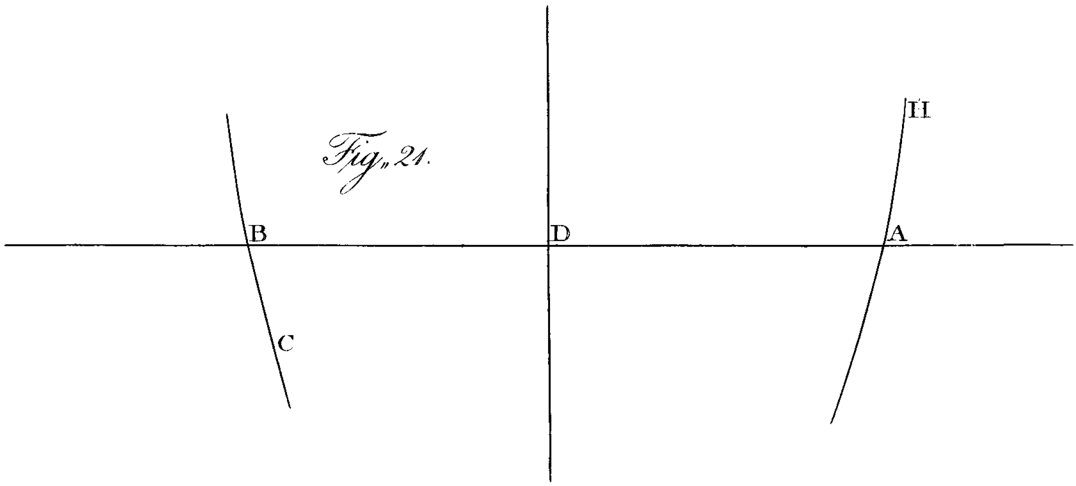


Fig. 21.

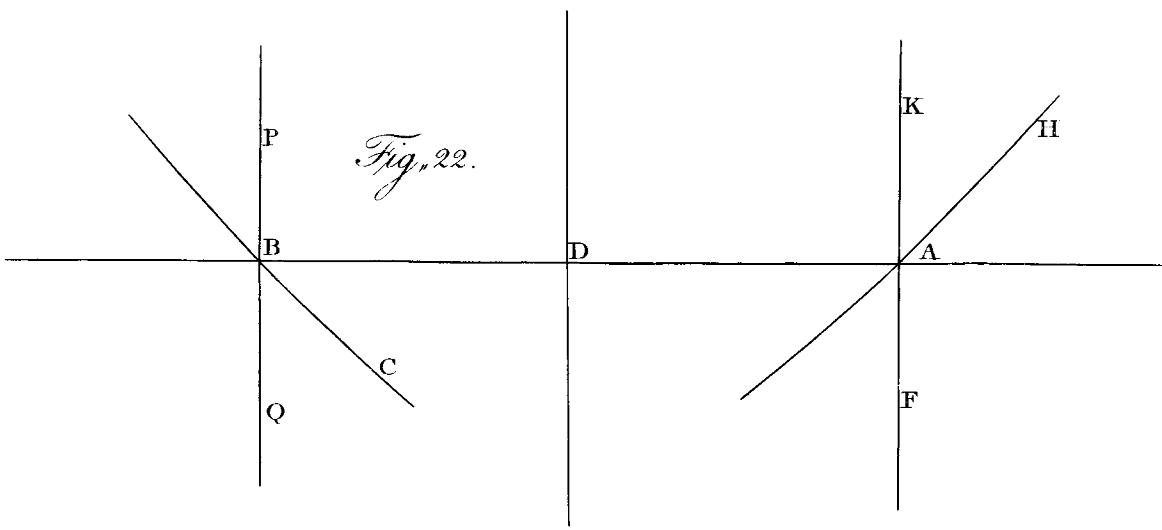


Fig. 22.

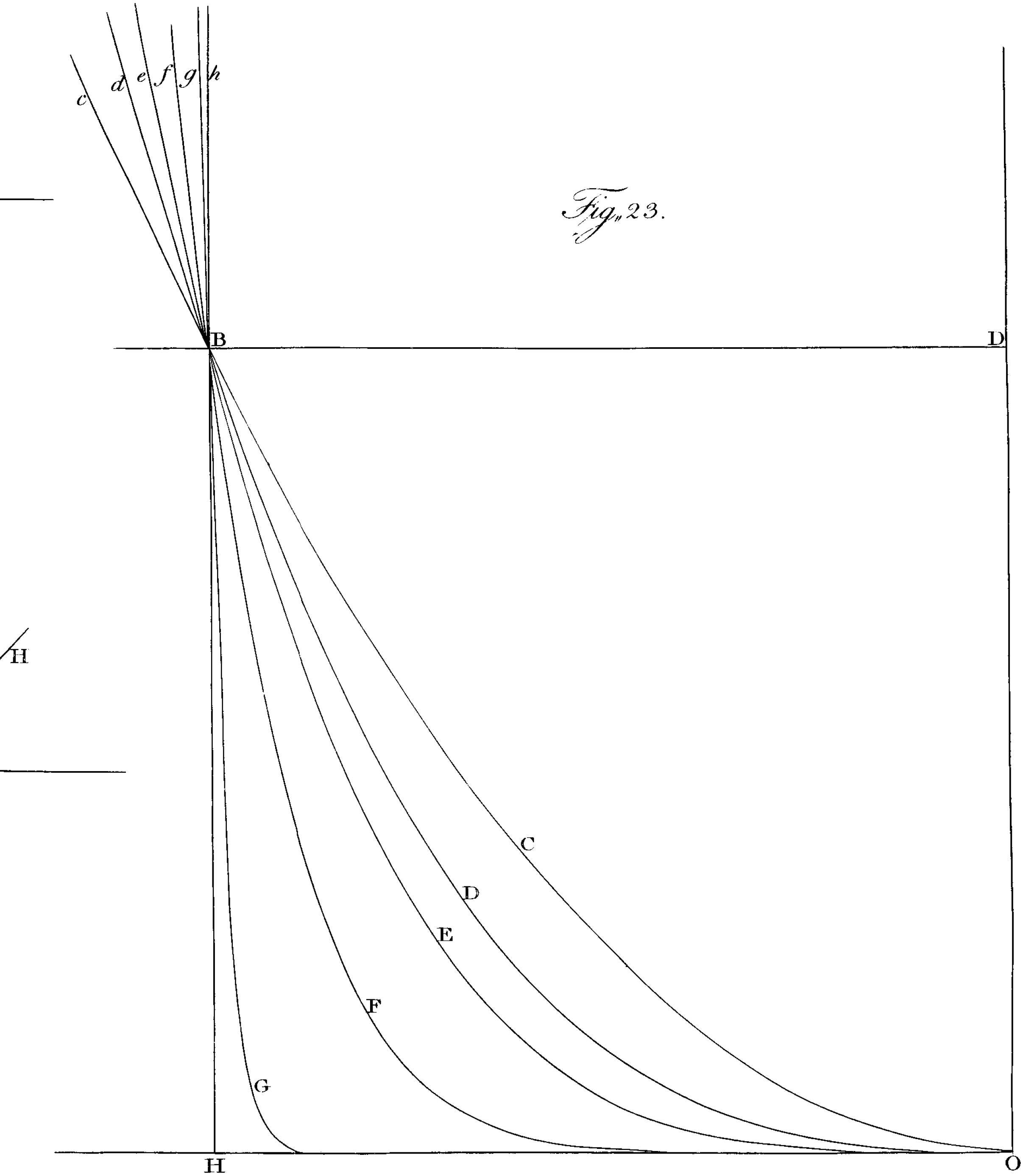


Fig. 23.

Fig. 24.

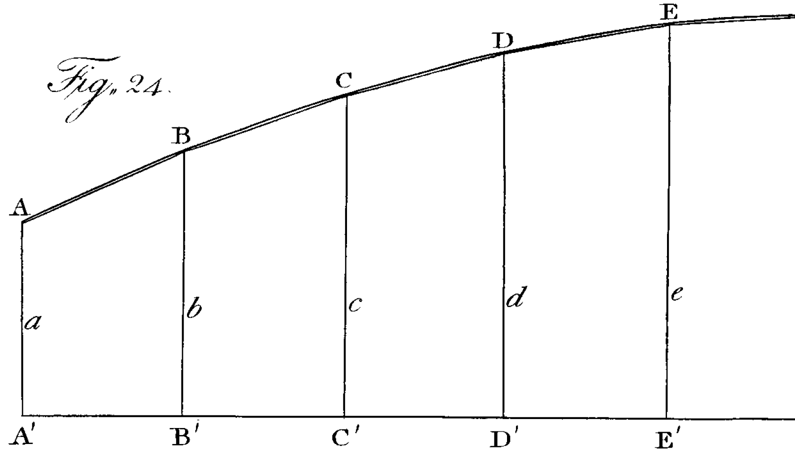
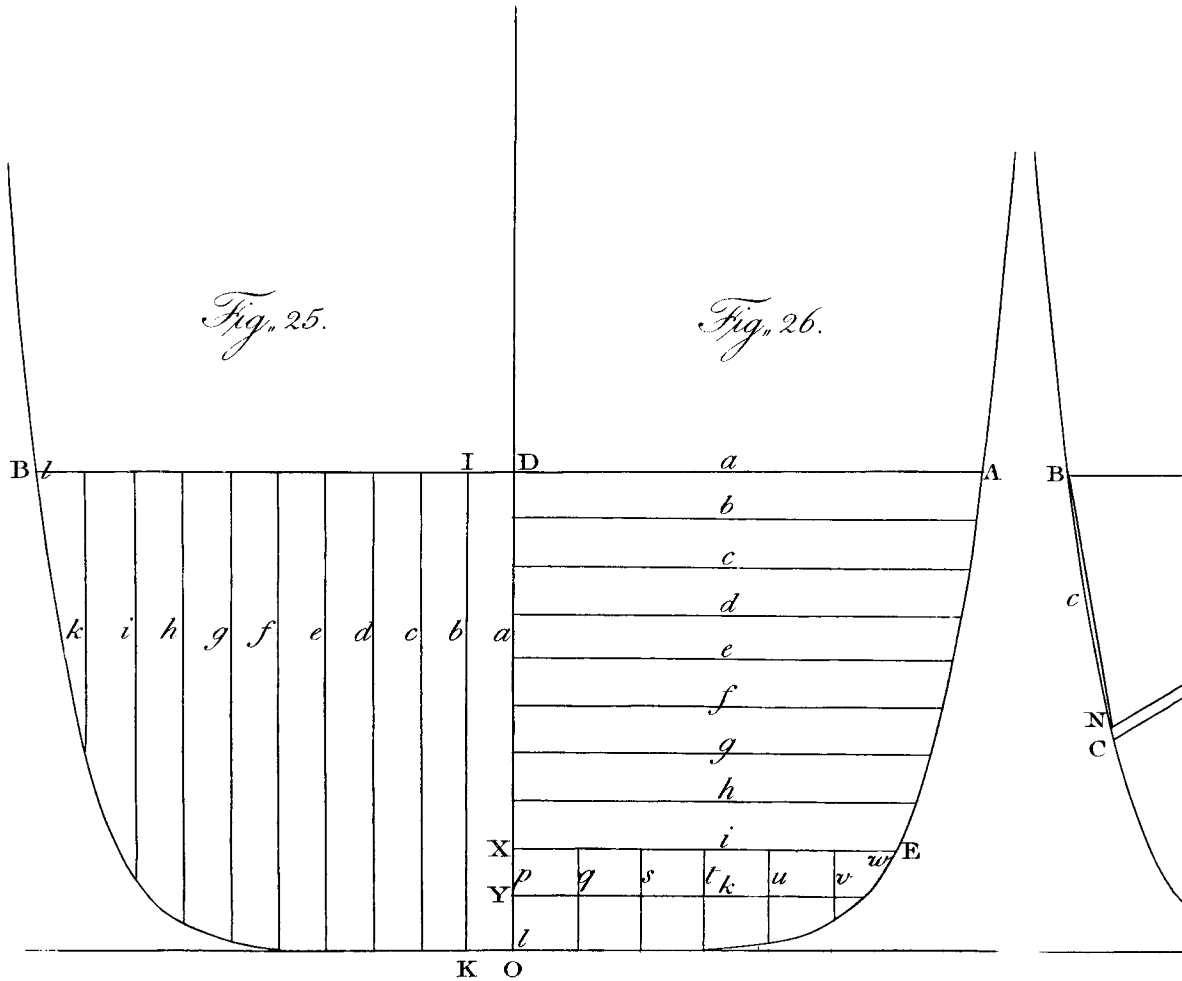


Fig. 25.

Fig. 26.



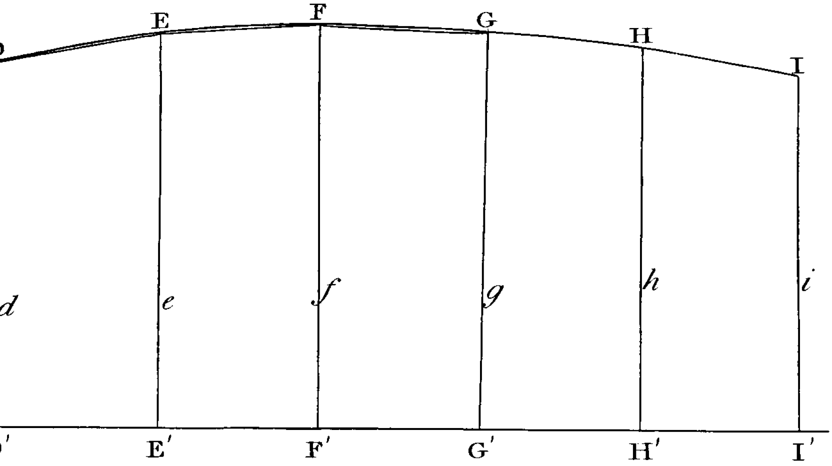
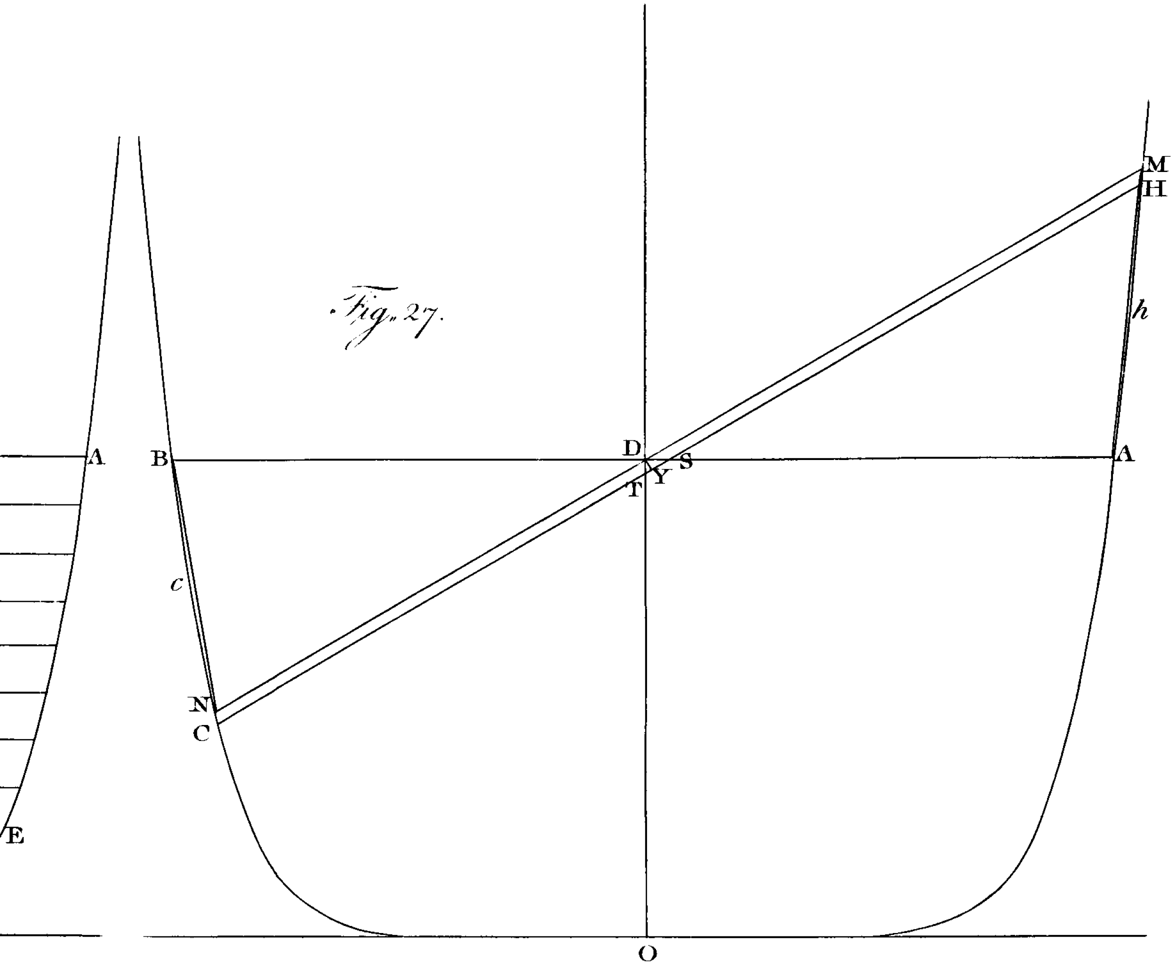


Fig. 27.



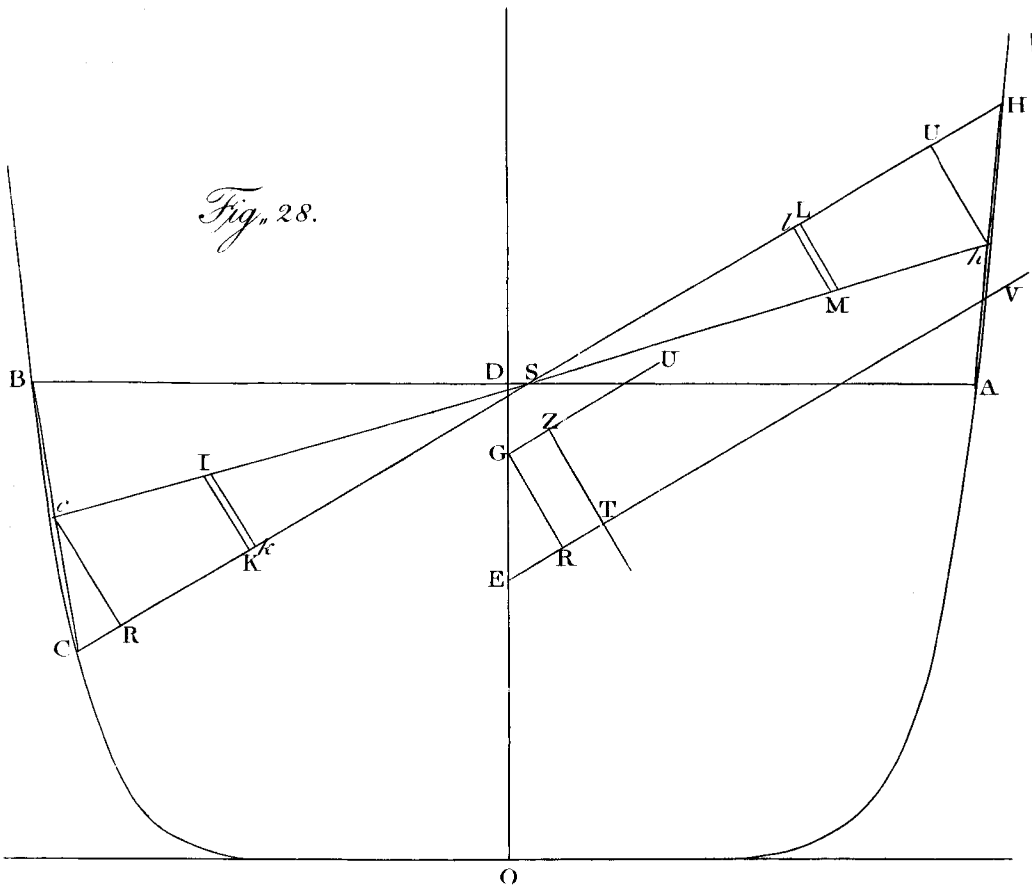
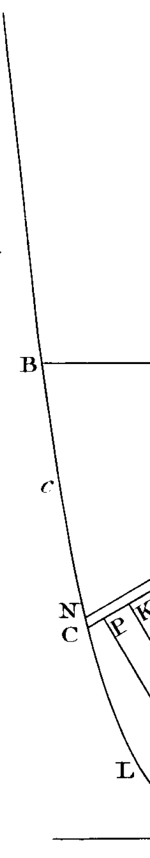


Fig. 28.



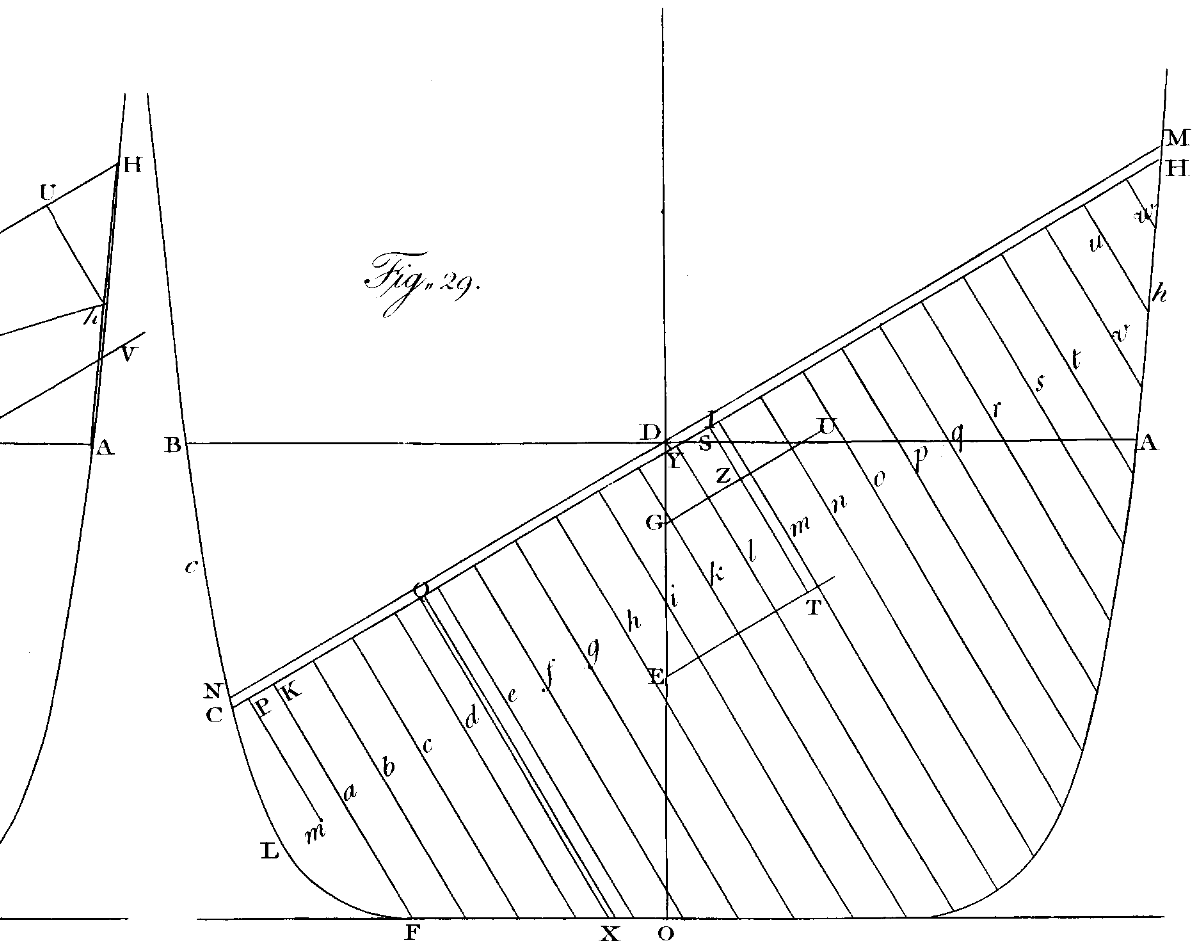

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Fig. 29.

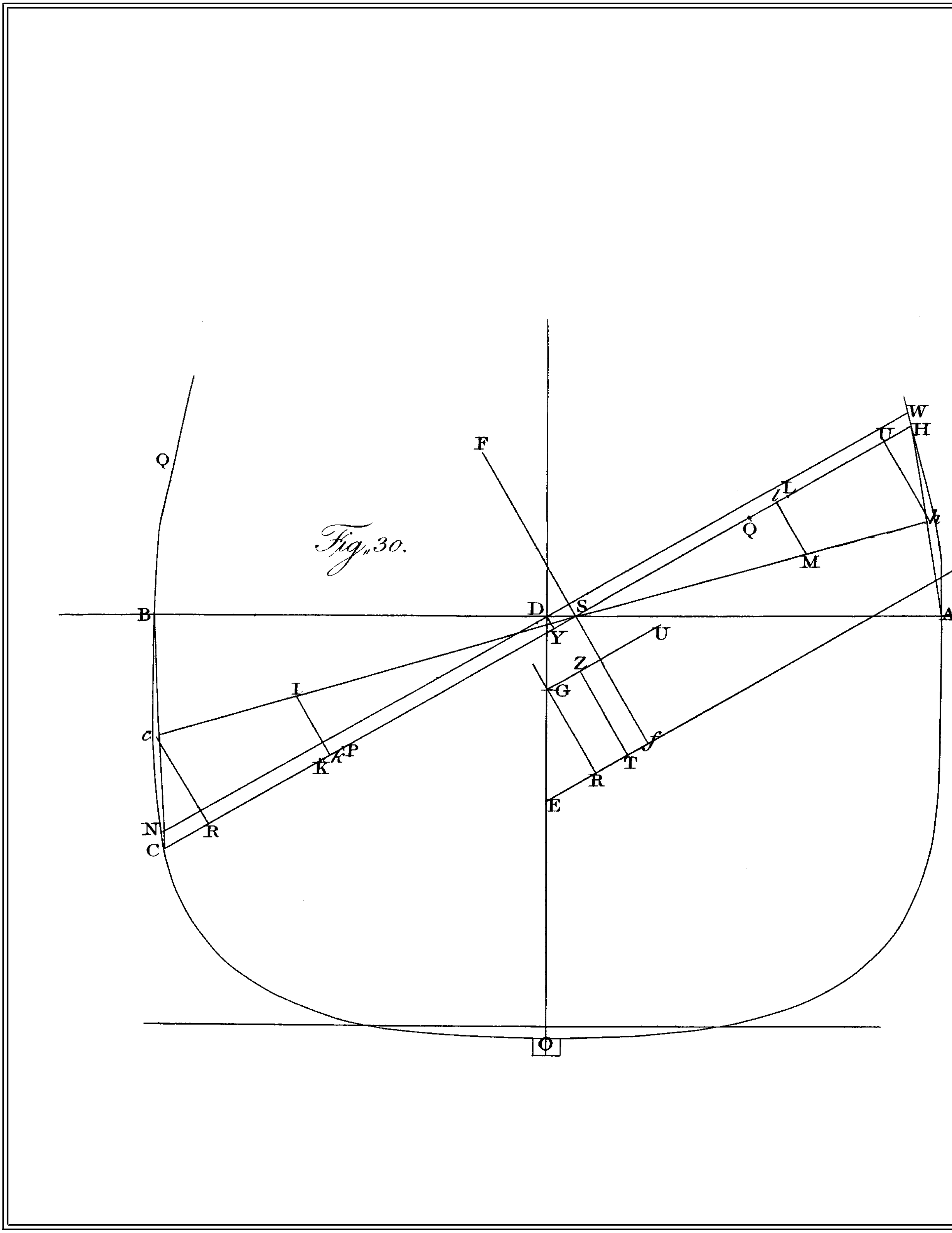

Fig. 30.
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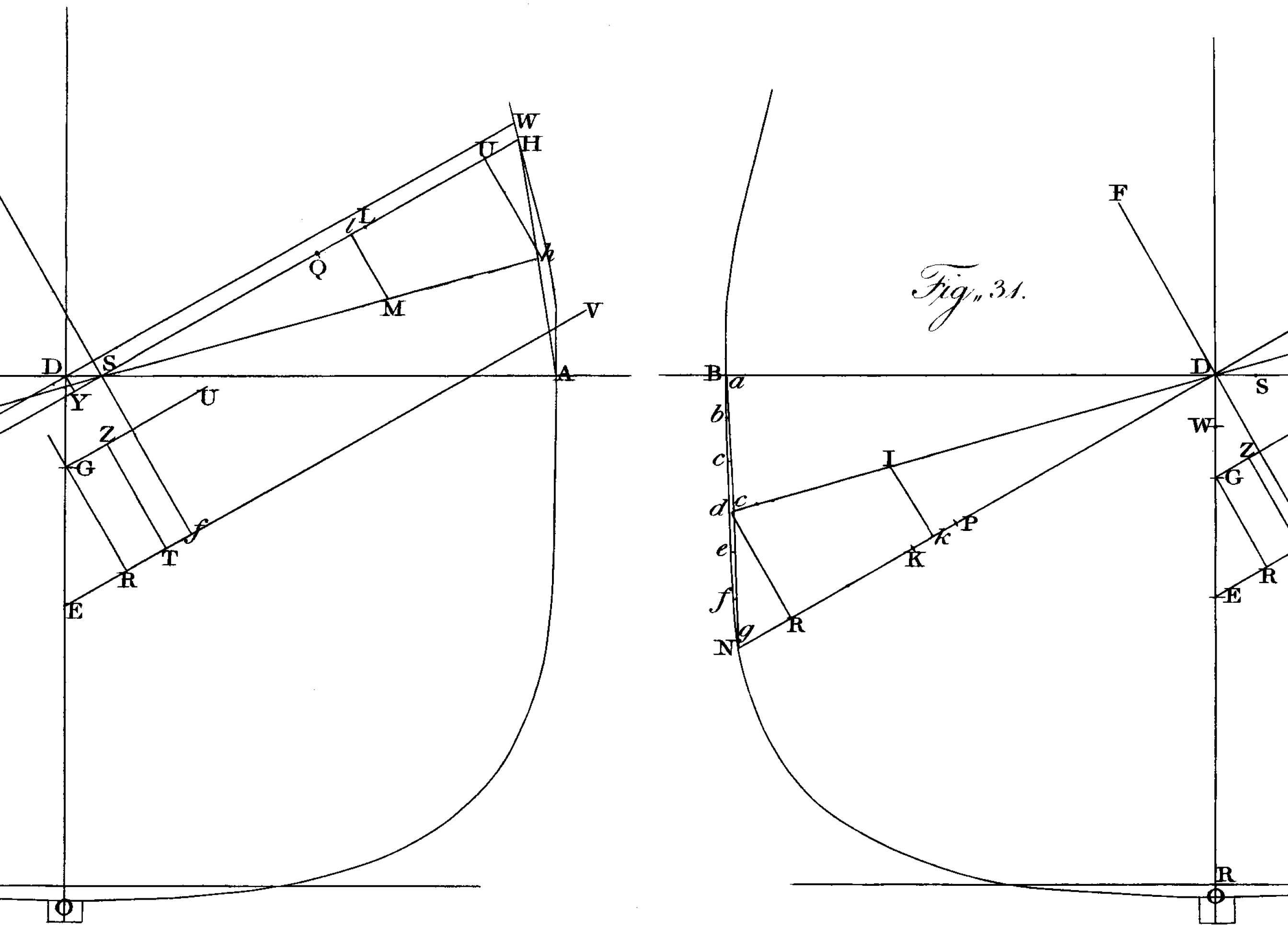

Fig. 31.

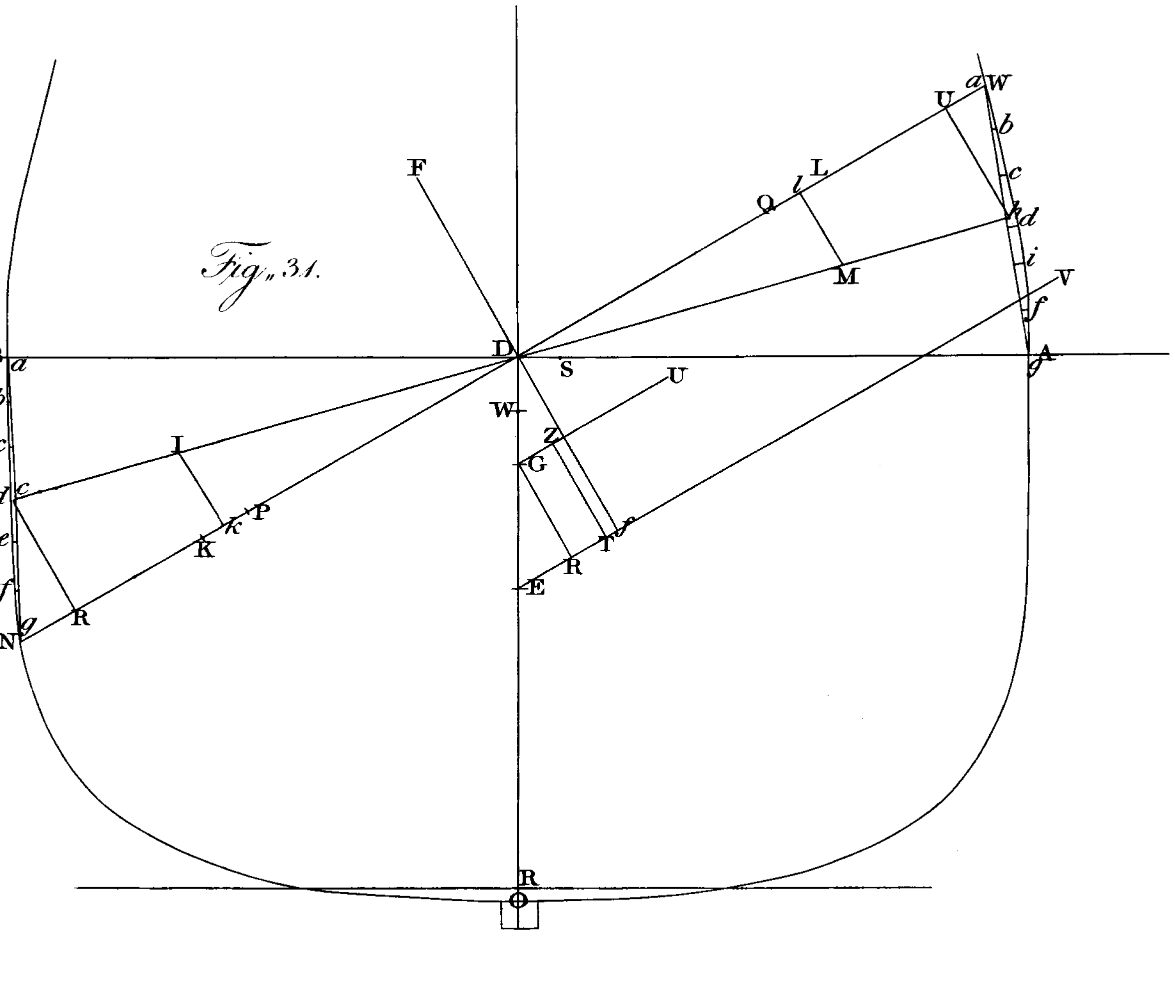

Fig. 31.

Note to the Investigation, Page 233.

$$WQ = FE \times \frac{\text{tang.}^2\tfrac{1}{2}F \times \text{tang. }S \times \text{sec. }S}{\sqrt{1 - \text{tang.}^2\tfrac{1}{2}F \times \text{tang.}^2 S}}$$

The investigation follows:

$FP : PO :: \text{sin. }O : \text{sin. }F$; wherefore $OP = FP \times \frac{\text{sin. }F}{\text{sin. }O}$; or because FP* $= FE \times \text{sec. }\tfrac{1}{2}F \times \sqrt{\frac{\text{sin. }O}{\text{sin. }P}}$, $OP = FE \times \frac{\text{sec. }\frac{1}{2}F \times \text{sin. }F}{\sqrt{\text{sin. }O \times \text{sin. }P}}$ $= FE \times \frac{2\,\text{sin. }\frac{1}{2}F}{\sqrt{\text{sin. }O \times \text{sin. }P}}$, and $\frac{OP}{2} = PQ = FE \times \frac{\text{sin. }\frac{1}{2}F}{\sqrt{\text{sin. }O \times \text{sin. }P}}$; or because $\sqrt{\text{sin. }O \times \text{sin. }P}$† $= \frac{\sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S}}{\text{sec. }\frac{1}{2}F \times \text{sec. }S}$, $PQ = FE \times \frac{\text{tang. }\frac{1}{2}F \times \text{sec. }S}{\sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang. }S}}$.

But $PW : WF$ ($= FE \times$‡ $\sqrt{1 - \text{tang.}^2\tfrac{1}{2}F \times \text{tang.}^2 S}$) $:: \text{sin. }\tfrac{1}{2}F : \text{sin. }P$; therefore $PW = FE \times \text{sin. }\tfrac{1}{2}F \times \frac{\sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S}}{\text{sin. }P}$

and $WQ = PW - PQ = FE \times \frac{\sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S} \times \text{sin. }\frac{1}{2}F}{\text{sin. }P}$ $- FE \times \frac{\text{tang. }\frac{1}{2}F \times \text{sec. }S}{\sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S}}$ or

$$WQ = FE \times \frac{\text{sin. }\frac{1}{2}F - \text{tang.}^2\frac{1}{2}F \times \text{sin. }\frac{1}{2}F \times \text{tang.}^2 S - \text{tang. }\frac{1}{2}F \times \text{sec. }S \times \text{sin. }P}{\text{sin. }P \times \sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S}}$$

But $\text{sin. }P = \text{cos. }\overline{S + \tfrac{1}{2}F} = \text{cos. }\tfrac{1}{2}F \times \text{cos. }S - \text{sin. }\tfrac{1}{2}F \times \text{sin. }S$, which being substituted for sin. P in the value of WQ just found, the result is

$$WQ = FE \times \frac{\text{sin. }\frac{1}{2}F - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S \times \text{sin. }\frac{1}{2}F - \text{sin. }\frac{1}{2}F + \text{tang. }\frac{1}{2}F \times \text{sin. }\frac{1}{2}F \times \text{tang. }S}{\text{cos. }\frac{1}{2}F \times \text{cos. }S - \text{sin. }\frac{1}{2}F \times \text{sin. }S \times \sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S}}$$

$$\text{or } WQ = FE \times \frac{\text{tang. }\frac{1}{2}F \times \text{sin. }\frac{1}{2}F \times \text{tang. }S - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S \times \text{sin. }\frac{1}{2}F}{\text{cos. }\frac{1}{2}F \times \text{cos. }S - \text{sin. }\frac{1}{2}F \times \text{sin. }S \times \sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S}}$$

$$\text{or } WQ = FE \times \frac{\text{tang. }\frac{1}{2}F \times \text{sin. }\frac{1}{2}F \times \text{tang. }S \times \overline{1 - \text{tang. }\frac{1}{2}F \times \text{tang. }S}}{\text{cos. }\frac{1}{2}F \times \text{cos. }S - \text{sin. }\frac{1}{2}F \times \text{sin. }S \times \sqrt{1 - \text{tang.}^2\frac{1}{2}F \times \text{tang.}^2 S}}, \text{ or}$$

* Demonstrated in page 232. † Page 308. ‡ Page 233.

because $\cos. \frac{1}{2} F \times \cos. S - \sin. \frac{1}{2} F \times \sin. S = \cos. \frac{1}{2} F \times \cos. S \times \overline{1 - \text{tang.} \frac{1}{2} F \times \text{tang.} S}$

$$WQ = FE \times \frac{\text{tang.} \frac{1}{2} F \times \sin. \frac{1}{2} F \times \text{tang.} S \times \overline{1 - \text{tang.} \frac{1}{2} F \times \text{tang.} S}}{\cos. \frac{1}{2} F \times \cos. S \times \overline{1 - \text{tang.} \frac{1}{2} F \times \text{tang.} S} \times \sqrt{\overline{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}}},$$

or $$WQ = FE \times \frac{\text{tang.}^2 \frac{1}{2} F \times \text{tang.} S \times \sec. S}{\sqrt{\overline{1 - \text{tang.}^2 \frac{1}{2} F \times \text{tang.}^2 S}}}.$$

The work of Mr. CHAPMAN, mentioned in page 267, is written in the Swedish language, and has been translated into French by M. VIAL DE CLAIRBOIS. The translation is intitled, *Traité de la Construction des Vaisseaux*, &c. *par* FREDERIC HENRI DE CHAPMAN, *Premier Constructeur des Armées Navales*, &c.

Dimensions

The Ordinates entered in the annexed Table are the

HORIZONTAL SECTIONS.

N°	1	2	3	4	5	6	7	8	9	10	11	12	13	14	1
	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Fe
16	13.5	17.1	18.9	19.85	20.4	20.55	20.65	20.55	20.5	20.5	20.5	20.5	20.5	20.5	20
15	13.0	17.0	18.9	20.0	20.55	20.75	20.9	21.0	21.0	21.0	21.0	21.0	21.0	21.0	21
14	12.4	16.75	18.8	20.05	26.6	20.9	21.1	21.2	21.3	21.3	21.3	21.3	21.3	21.3	21
13	11.6	16.4	18.6	20.0	20.16	20.105	21.15	21.3	21.4	21.45	21.45	21.45	21.45	21.45	21
12	10.78	16.0	18.4	19.85	20.54	20.94	21.2	21.38	21.48	21.5	21.56	21.56	21.56	21.56	21
11	9.6	15.35	18.1	19.7	20.5	20.92	21.21	21.44	21.55	21.60	21.60	21.60	21.60	21.60	21
10	8.1	14.4	17.7	19.45	20.3	20.8	21.15	21.38	21.52	21.60	21.60	21.60	21.60	21.60	21
9	6.6	13.25	17.075	19.1	20.075	20.65	21.05	21.34	21.45	21.5	21.59	21.59	21.59	21.59	21
8	4.9	11.73	16.24	18.55	19.73	20.45	20.9	21.2	21.34	21.47	21.51	21.51	21.51	21.51	21
7	3.3	10.0	15.1	17.84	19.3	20.12	20.65	20.9	21.14	21.28	21.51	21.51	21.51	21.35	21
6	1.7	8.2	13.5	16.85	18.64	19.65	20.32	20.62	20.78	20.93	20.93	21.1	21.1	21.1	21
5		6.2	11.3	15.3	17.55	18.91	19.60	20.05	20.25	20.4	20.5	20.6	20.6	20.60	20
4		4.09	9.0	13.2	15.09	17.7	18.65	19.05	19.35	19.53	19.6	19.75	19.75	19.75	19
3		1.8	6.35	10.45	13.5	15.6	16.8	17.45	17.88	18.15	18.3	18.3	18.3	18.3	18
2			3.5	6.85	9.78	12.15	13.6	14.55	15.2	15.5	15.9	15.9	15.9	15.9	15
1				2.55	3.09	5.5	6.75	8.	9.1	9.8	10.5	10.5	10.5	10.5	10

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Dimensions of the CUFFNELLS INDIA SHIP.

tes entered in the annexed Table are the Half-breadths, expressed in Feet, of the several vertical and horiz

VERTICAL SECTIONS.

9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.5	20.4	20.3	20.2	20.15	19.85	19.75	19.5
21.0	21.0	21.0	21.0	21.0	21.0	21.0	21.0	21.0	21.0	20.9	20.8	20.65	20.5	20.3	20.1	19.9
21.3	21.3	21.3	21.3	21.3	21.3	21.3	21.3	21.3	21.3	21.25	21.1	21.0	20.85	20.65	20.45	20.25
21.4	21.45	21.45	21.45	21.45	21.45	21.45	21.45	21.45	21.45	21.4	21.25	21.15	21.0	20.8	20.6	20.4
21.48	21.5	21.56	21.56	21.56	21.56	21.56	21.55	21.53	21.51	21.48	21.32	21.22	21.05	20.82	20.61	20.44
21.55	21.60	21.60	21.60	21.60	21.60	21.60	21.60	21.60	21.52	21.50	21.35	21.24	21.075	20.82	20.61	20.43
21.52	21.60	21.60	21.60	21.60	21.60	21.60	21.60	21.60	21.55	21.50	21.35	21.25	21.05	20.83	20.61	20.4
21.45	21.5	21.59	21.59	21.59	21.59	21.59	21.59	21.55	21.52	21.44	21.3	21.18	21.0	20.75	20.55	20.3
21.34	21.47	21.51	21.51	21.51	21.51	21.51	21.51	21.50	21.4	21.3	21.2	21.05	20.89	20.65	20.4	20.1
21.14	21.28	21.51	21.51	21.51	21.35	21.35	21.32	21.3	21.2	21.1	21.0	21.85	20.68	20.4	20.15	19.85
20.78	20.93	20.93	21.1	21.1	21.1	21.1	21.05	21.0	20.9	20.81	20.67	20.5	20.32	20.05	19.75	19.3
20.25	20.4	20.5	20.6	20.6	20.60	20.6	20.52	20.45	20.35	20.25	20.1	19.9	19.7	19.3	19.0	18.46
19.35	19.53	19.6	19.75	19.75	19.75	19.75	19.65	19.55	19.45	19.3	19.15	18.9	18.62	18.28	17.75	17.1
17.88	18.15	18.3	18.3	18.3	18.3	18.3	18.20	18.05	17.95	17.75	17.52	17.25	16.9	16.40	15.7	14.8
15.2	15.5	15.9	15.9	15.9	15.9	15.9	15.7	15.5	15.35	15.0	14.6	14.2	13.62	12.9	11.9	10.8
9.1	9.8	10.5	10.5	10.5	10.5	10.5	10.3	9.8	9.2	8.5	8.	7.2	6.4	5.9	5.1	4.2

Common distance between the vertical sections, 5 feet.

Common distance between the water-lines or horizontal sections, 2 feet.

Distance between the horizontal section 1, and the upper surface of the keel, $\frac{3}{4}$ of a foot.

The ordinates of the horizontal section 12 coincide with the water's surface when the vessel is loaded.

[To face page 310.

INDIA SHIP.

in Feet, of the several vertical and horizontal Sections.

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9	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
et.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
4	20.3	20.2	20.15	19.85	19.75	19.5	19.3	19.1	18.9	18.5	18.05	17.5	16.95	16.25	15.5
9	20.8	20.65	20.5	20.3	20.1	19.9	19.7	19.45	19.3	18.8	18.4	17.9	17.35	16.65	15.7
25	21.1	21.0	20.85	20.65	20.45	20.25	20.0	19.7	19.4	19.05	18.7	18.2	17.6	16.9	15.8
4	21.25	21.15	21.0	20.8	20.6	20.4	20.1	19.8	19.5	19.25	18.75	18.35	17.6	16.9	15.5
.48	21.32	21.22	21.05	20.82	20.61	20.44	20.15	19.85	19.58	19.25	18.77	18.21	17.52	16.5	12.95
50	21.35	21.24	21.075	20.82	20.61	20.43	20.12	19.8	19.52	19.15	18.62	17.9	16.9	14.9	7.
.50	21.35	21.25	21.05	20.83	20.61	20.4	20.1	19.75	19.44	19.	18.4	17.4	15.65	14.5	3.4
.44	21.3	21.18	21.0	20.75	20.55	20.3	20.0	19.59	19.22	18.55	17.82	16.3	13.4	7.45	1.95
3	21.2	21.05	20.89	20.65	20.4	20.1	19.73	19.3	18.8	17.9	16.8	14.6	10.35	4.31	1.35
1	21.0	21.85	20.68	20.4	20.15	19.85	19.35	18.8	18.1	16.85	15.05	12.0	7.1	2.7	1.05
.81	20.67	20.5	20.32	20.05	19.75	19.3	18.8	18.05	17.11	15.35	12.9	9.1	4.55	1.85	.90
.25	20.1	19.9	19.7	19.3	19.0	18.46	17.88	16.8	15.45	13.10	10.1	6.12	2.9	1.32	.75
.3	19.15	18.9	18.62	18.28	17.75	17.1	16.15	14.8	12.85	10.1	7.05	3.75	1.9	1.0	.70
75	17.52	17.25	16.9	16.40	15.7	14.8	13.4	11.6	9.35	6.65	4.25	2.3	1.25	.80	.65
.0	14.6	14.2	13.62	12.9	11.9	10.8	9.2	7.2	5.38	3.55	2.4	1.45	.98	.75	.63
5	8.	7.2	6.4	5.9	5.1	4.2	3.35	2.5	1.8	1.40	1.11	.90	.80	.62	.60

2 feet.
of the keel, ¼ of a foot.
er's surface when the vessel is loaded.